

PROBLEMAS RESUELTOS DE INTRODUCCIÓN Y VECTORES

① a)

$$300 \cancel{\text{dm}} \cdot \frac{1 \text{ km}}{10^4 \cancel{\text{dm}}} = \frac{300}{10^4} = \boxed{0.3 \text{ km}}$$

$$b) 400 \cancel{\text{cg}^2} \cdot \frac{1 \text{ dag}^2}{10^6 \cancel{\text{cg}^2}} = \frac{400}{10^6} = \boxed{4 \cdot 10^{-4} \text{ dag}^2}$$

$$c) 40 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \cdot \frac{1000 \cancel{\text{m}}}{1 \cancel{\text{km}}} \cdot \frac{1 \cancel{\text{K}}}{3600 \cancel{\text{s}}} = \frac{40000}{3600} = \boxed{11.1 \frac{\text{m}}{\text{s}}}$$

$$\textcircled{2} a) 8 \cancel{\text{K}} \cdot \frac{3600 \cancel{\text{s}}}{1 \cancel{\text{h}}} \cdot \frac{10^3 \cancel{\text{ms}}}{1 \cancel{\text{s}}} = \boxed{2.88 \cdot 10^7 \text{ ms}}$$

$$b) 50 \cancel{\text{ml}} \cdot \frac{1 \cancel{\text{cm}^3}}{1 \cancel{\text{ml}}} \cdot \frac{1 \text{ dam}^3}{10^9 \cancel{\text{cm}^3}} = \frac{50}{10^9} = \boxed{5 \cdot 10^{-8} \text{ dam}^3}$$

$$\textcircled{3} a) 5 \cancel{\text{mg}} \cdot \frac{1 \cancel{\text{g}}}{10^3 \cancel{\text{mg}}} \cdot \frac{1 \text{ Gg}}{10^9 \cancel{\text{g}}} = \boxed{5 \cdot 10^{-12} \text{ Gg}}$$

$$b) 400 \cancel{\text{km}^2} \cdot \frac{10^{10} \cancel{\text{cm}^2}}{1 \cancel{\text{km}^2}} = \boxed{4 \cdot 10^{12} \text{ cm}^2}$$

$$c) 800 \cancel{\text{cm}^3} \cdot \frac{1 \text{ dam}^3}{10^9 \cancel{\text{cm}^3}} = \boxed{8 \cdot 10^{-7} \text{ dam}^3}$$

$$\textcircled{4} \text{ a) } 200 \cancel{\text{ m}^3} \cdot \frac{1 \cancel{\text{ cm}^3}}{1 \cancel{\text{ m}^3}} \cdot \frac{1 \text{ km}^3}{10^{12} \cancel{\text{ cm}^3}} = \boxed{2 \cdot 10^{-10} \text{ km}^3}$$

$$\text{b) } 9000 \cancel{\text{ Tm}^2} \cdot \frac{10^{36} \cancel{\text{ m}^3}}{1 \cancel{\text{ Tm}^3}} \cdot \frac{1000 \cancel{\text{ l}}}{1 \cancel{\text{ m}^3}} \cdot \frac{100 \text{ cl}}{1 \cancel{\text{ l}}} = \boxed{9 \cdot 10^{44} \text{ cl}}$$

$$\textcircled{5} \text{ a) } 1 \cancel{\text{ a}^\circ} \cdot \frac{365 \cancel{\text{ d}^\circ\text{a}}}{1 \cancel{\text{ a}^\circ}} \cdot \frac{24 \cancel{\text{ h}}}{1 \cancel{\text{ d}^\circ\text{a}}} \cdot \frac{3600 \text{ s}}{1 \cancel{\text{ h}}} = \boxed{315 \cdot 10^7 \text{ s}}$$

$$\text{b) } 1 \text{ a}^\circ\text{-luz} = 300\,000 \frac{\text{km}}{\text{s}} \cdot 1 \cancel{\text{ a}^\circ} \cdot \frac{365 \cancel{\text{ d}^\circ\text{a}}}{1 \cancel{\text{ a}^\circ}} \cdot \frac{24 \cancel{\text{ h}}}{1 \cancel{\text{ d}^\circ\text{a}}}$$

$$\cdot \frac{3600 \text{ s}}{1 \cancel{\text{ h}}} = \boxed{946 \cdot 10^{12} \text{ km}}$$

$$\textcircled{6} \text{ a) } 500 \frac{\cancel{\text{ kg}^2}}{\cancel{\text{ cm} \cdot \text{s}^3}} \cdot \frac{10^{12} \text{ mg}^2}{1 \cancel{\text{ kg}^2}} \cdot \frac{10^3 \cancel{\text{ cm}}}{1 \text{ dam}} \cdot \frac{3600^3 \cancel{\text{ s}^3}}{1 \text{ h}^3} =$$

$$= \boxed{2'33 \cdot 10^{28} \frac{\text{mg}^2}{\text{dam} \cdot \text{h}^3}}$$

$$\text{b) } 6 \frac{\cancel{\text{ mm}^2}}{\cancel{\text{ kg} \cdot \text{min}^4}} \cdot \frac{1 \cancel{\text{ km}^2}}{10^{12} \cancel{\text{ mm}^2}} \cdot \frac{1 \cancel{\text{ kg}}}{10^9 \mu\text{g}} \cdot \frac{1 \cancel{\text{ min}^4}}{60^4 \cancel{\text{ s}^4}} =$$

$$= \boxed{4'63 \cdot 10^{-28} \frac{\text{km}^2}{\mu\text{g} \cdot \text{s}^4}}$$

$$\textcircled{7} \text{ a) } 20 \frac{\cancel{\mu\text{L}^2}}{\cancel{\text{km} \cdot \text{ms}^2}} \cdot \frac{1 \cancel{\text{km}}}{10^2 \text{ dam}} \cdot \frac{10^{-12} \cancel{\text{L}^2}}{1 \cancel{\mu\text{L}^2}} \cdot \frac{10^4 \text{ cL}^2}{1 \cancel{\text{L}^2}} \cdot \frac{10^6 \cancel{\text{ms}^2}}{1 \cancel{\text{s}^2}} \cdot \frac{60^2 \cancel{\text{s}^2}}{1 \text{ min}^2} = \boxed{72 \frac{\text{cL}^2}{\text{dam} \cdot \text{min}^2}}$$

$$\text{b) } 12 \frac{\cancel{\text{cm}^3}}{\cancel{\mu\text{g}} \cdot \text{s}^4} \cdot \frac{1 \text{ m}^3}{10^6 \cancel{\text{cm}^3}} \cdot \frac{10^{12} \cancel{\mu\text{g}}}{1 \cancel{\text{g}}} \cdot \frac{1 \cancel{\text{g}}}{10^6 \mu\text{g}} = \boxed{12 \frac{\text{m}^3}{\mu\text{g} \cdot \text{s}^4}}$$

$$\textcircled{8} \quad \mu = \frac{F \cdot h}{A \cdot v} \Rightarrow [\mu] = \frac{[F] \cdot [h]}{[A] \cdot [v]} = \frac{\frac{\text{N} \cdot \cancel{\text{K}}}{\cancel{\text{T}^2}} \cdot \cancel{\text{K}}}{\cancel{\text{K}^2} \cdot \frac{\text{L}}{\cancel{\text{T}}}} = \frac{\text{N}}{\text{L} \cdot \text{T}} \rightarrow \boxed{\frac{\text{kg}}{\text{m} \cdot \text{s}}}$$

$$\textcircled{9} \quad [E] = \left[\frac{1}{2} \cdot m \cdot v^2 \right] = [m \cdot g \cdot h]$$

$$\frac{\text{M} \cdot \text{L}^2}{\text{T}^2} = \text{M} \cdot \frac{\text{L}^2}{\text{T}^2} = \text{M} \cdot \frac{\text{L}}{\text{T}^2} \cdot \text{L}$$

$$\frac{\text{M} \cdot \text{L}^2}{\text{T}^2} = \frac{\text{M} \cdot \text{L}^2}{\text{T}^2} = \frac{\text{M} \cdot \text{L}^2}{\text{T}^2}$$

$$\textcircled{10} \quad G = \frac{F \cdot r^2}{M \cdot m} \Rightarrow [G] = \frac{[F] \cdot [r]^2}{[M] \cdot [m]} =$$

$$= \frac{\frac{M \cdot L}{T^2} \cdot L^2}{M \cdot M} = \frac{L^3}{M \cdot T^2} \rightarrow \boxed{\frac{m^3}{kg \cdot s^2}}$$

$$\textcircled{11} \quad [Pot] = \frac{M \cdot L^2}{T^3} ; [a] = \frac{L}{T^2} \Rightarrow$$

$$\Rightarrow \frac{[Pot]}{[a]} = \frac{\frac{M \cdot L^2}{T^3}}{\frac{L}{T^2}} = M \cdot \frac{L}{T} = M \cdot [v] \Rightarrow$$

$$\Rightarrow \boxed{Pot = m \cdot v \cdot a}$$

$$\textcircled{12} \quad [F] = \frac{M \cdot L}{T^2} , [Pot] = \frac{M \cdot L^2}{T^3} \Rightarrow$$

$$\Rightarrow \frac{[Pot]}{[F]} = \frac{\frac{M \cdot L^2}{T^3}}{\frac{M \cdot L}{T^2}} = \frac{L}{T} = [v] \Rightarrow \boxed{Pot = F \cdot v}$$

$$(13) \quad \bar{x} = \frac{\sum x_i}{N} = \frac{10'15 + 10'17 + 10'18 + 10'17}{4} = 10'17$$

$$\varepsilon_{a3} = |x - \bar{x}| = |10'18 - 10'17| = 0'01$$

$$\varepsilon_{r3} = \frac{\varepsilon_{a3} \cdot 100}{\bar{x}} = \frac{0'01 \cdot 100}{10'17} = \boxed{0'0983 \%}$$

$$(14) \quad \bar{x} = \frac{\sum x_i}{N} = \frac{1'23 + 1'28 + 1'29 + 1'35 + 1'27}{5} = 1'28$$

$$\varepsilon_a = |x - \bar{x}|; \quad \varepsilon_a = 0'05, 0, 0'01, 0'07, 0'01$$

$$\delta = \frac{\sum \varepsilon_{ai}}{N} = \frac{0'05 + 0 + 0'01 + 0'07 + 0'01}{5} = 0'028 \approx 0'03$$

$$\boxed{1'28 \pm 0'03}$$

$$(15) \quad \bar{x} = \frac{\sum x_i}{N} = \frac{4'56 + 4'57 + 4'52 + 4'58 + 4'59 + 4'55}{6} = 4'56$$

$$\varepsilon_a = |\bar{x} - x| = |4'56 - 4'52| = \boxed{0'04}$$

$$(16) \quad \bar{x} = \frac{\sum x_i}{N} = \frac{24 + 23 + 26 + 25 + 22 + 23 + 28}{7} = 24'43 \approx 24$$

$$\varepsilon_{a2} = |\bar{x} - x| = |24 - 23| = 1$$

$$\varepsilon_{r2} = \frac{\varepsilon_{a2} \cdot 100}{\bar{x}} = \frac{1 \cdot 100}{24} = \boxed{4'17 \%}$$

$$(17) \quad \bar{x} = \frac{\sum x_i}{N} = \frac{67'23 + 67'31 + 67'42 + 67'38 + 67'62 + 67'56}{6} =$$

$$= 67'42$$

$$\varepsilon_a = |\bar{x} - x_i|$$

x_i	67'23	67'31	67'42	67'38	67'62	67'56
ε_a	0'19	0'11	0	0'04	0'20	0'14

$$\delta = \frac{\sum \varepsilon_{ai}}{N} = \frac{0'19 + 0'11 + 0 + 0'04 + 0'20 + 0'14}{6} = 0'1133 \approx 0'11$$

$$\boxed{67'42 \pm 0'11}$$

(18) La de menor error relativo:

$$\varepsilon_{r1} = \frac{\varepsilon_{a1} \cdot 100}{\bar{x}_1} = \frac{0'5 \cdot 100}{2'1} = 23'8\%$$

$$\varepsilon_{r2} = \frac{\varepsilon_{a2} \cdot 100}{\bar{x}} = \frac{0'1 \cdot 100}{1'80} = 5'56\% . \quad \boxed{\text{La segunda}}$$

$$(19) \quad \varepsilon_r = \frac{\varepsilon_a \cdot 100}{\bar{x}}$$

$$\varepsilon_a = |\bar{x} - x| = |9'8 - 10| = 0'2 \quad \frac{m}{s^2}$$

$$\varepsilon_r = \frac{0'2 \cdot 100}{9'8} = \boxed{2'04\%}$$

(20) a) Cuatro lugares a la derecha:

$$4'346789 \cdot 10^{-4}$$

b) Movemos la coma tres lugares a la izquierda:

$$8'635475 \cdot 10^3$$

(21) a) $8'82394 \cdot 10^{-3} \simeq 8'82 \cdot 10^{-3}$

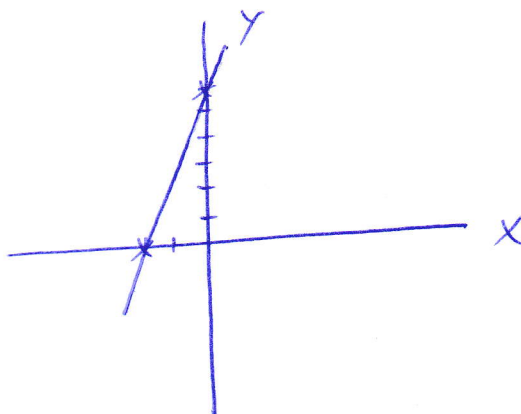
b) $6'54567 \cdot 10^5 \simeq 6'55 \cdot 10^5$

(22) a) $\frac{3'27}{3} + \frac{8'4}{2} = 11'67 \simeq \boxed{\frac{12}{2}}$

b) $\frac{\overbrace{6'243}^4 \cdot \overbrace{8'57}^3}{\underbrace{45'5623}_6} = 1'174271... \simeq \boxed{\frac{1'17}{3}}$

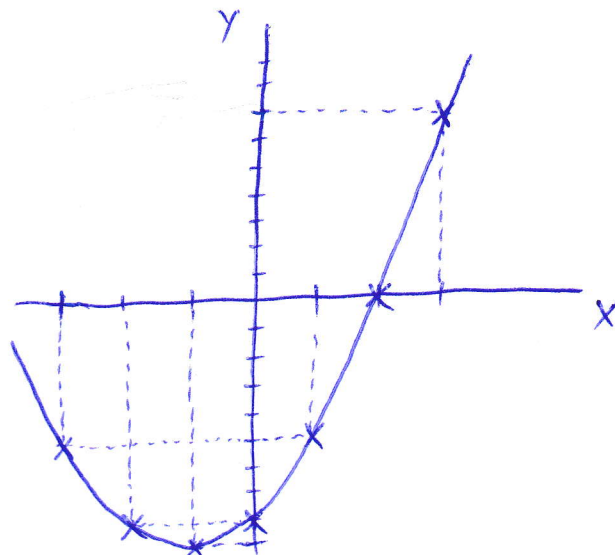
(23)

x	y
0	6
-2	0



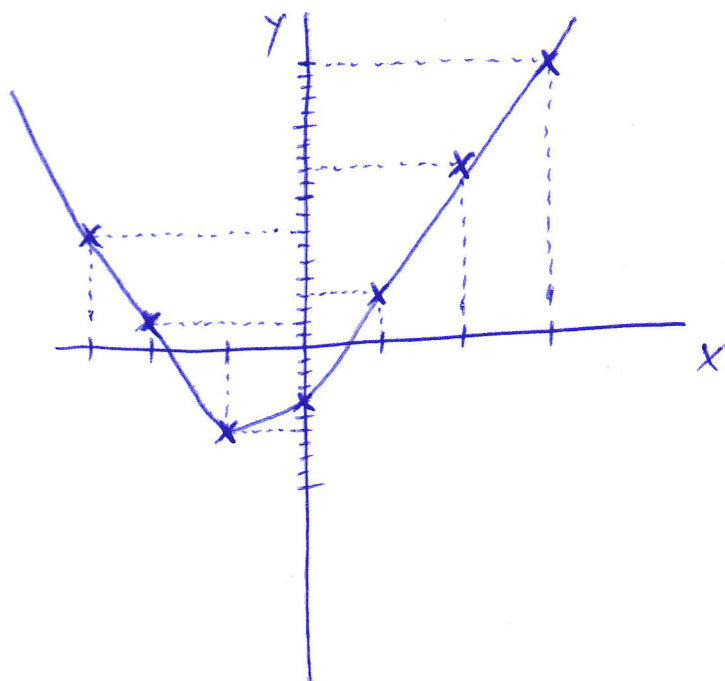
24) $y = x^2 + 2x - 8$

x	y
-3	$(-3)^2 + 2 \cdot (-3) - 8 = 9 - 6 - 8 = -5$
-2	$(-2)^2 + 2 \cdot (-2) - 8 = 4 - 4 - 8 = -8$
-1	$(-1)^2 + 2 \cdot (-1) - 8 = 1 - 2 - 8 = -9$
0	$0^2 + 2 \cdot 0 - 8 = 0 + 0 - 8 = -8$
1	$1^2 + 2 \cdot 1 - 8 = 1 + 2 - 8 = -5$
2	$2^2 + 2 \cdot 2 - 8 = 4 + 4 - 8 = 0$
3	$3^2 + 2 \cdot 3 - 8 = 9 + 6 - 8 = 7$



25) $y = 3x^2 + 5x - 4$

x	y
-3	$3 \cdot (-3)^2 + 5 \cdot (-3) - 4 = 8$
-2	$3 \cdot (-2)^2 + 5 \cdot (-2) - 4 = -2$
-1	$3 \cdot (-1)^2 + 5 \cdot (-1) - 4 = -6$
0	$3 \cdot 0 + 5 \cdot 0 - 4 = -4$
1	$3 \cdot 1 + 5 \cdot 1 - 4 = 4$
2	$3 \cdot 2 + 5 \cdot 2 - 4 = 12$
3	$3 \cdot 3 + 5 \cdot 3 - 4 = 20$



26) $y = mx + n$; $n = 10$

$$m = \frac{\Delta y}{\Delta x} = \frac{40 - 10}{20 - 0} = \frac{30}{20} = 1.5$$

$y = 1.5 \cdot x + 10$

27) $y = m \cdot x + n ; n = 40$

$$m = \frac{\Delta y}{\Delta x} = \frac{20 - 40}{20 - 0} = -\frac{20}{20} = -1 ; \boxed{y = -x + 40}$$

28) a) $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{2^2 + 3^2 + (-4)^2} = \boxed{5,39}$

b) $\cos \alpha = \frac{A_x}{A} = \frac{2}{5,39} = 0,371 \Rightarrow \alpha = \boxed{68,2^\circ}$

c) $\cos \beta = \frac{A_y}{A} = \frac{3}{5,39} = 0,557 \Rightarrow \beta = \boxed{56,2^\circ}$

d) $\vec{u}_A = \frac{\vec{A}}{A} = \frac{2\vec{i} + 3\vec{j} - 4\vec{k}}{5,39} = \frac{2}{5,39} \cdot \vec{i} + \frac{3}{5,39} \cdot \vec{j} - \frac{4}{5,39} \cdot \vec{k} =$
 $= \boxed{0,371 \cdot \vec{i} + 0,557 \cdot \vec{j} - 0,742 \cdot \vec{k}}$

29) a) $\vec{r} + \vec{v} = (2-4) \cdot \vec{i} + (3+6) \cdot \vec{j} + (-4-2) \cdot \vec{k} =$
 $= \boxed{-2 \cdot \vec{i} + 9 \cdot \vec{j} - 6 \cdot \vec{k}}$

b) $\vec{r} - \vec{v} = (2+4) \cdot \vec{i} + (3-6) \cdot \vec{j} + (-4+2) \cdot \vec{k} =$
 $= \boxed{6 \cdot \vec{i} - 3 \cdot \vec{j} - 2 \cdot \vec{k}}$

c) $\vec{v} - \vec{r} = -(\vec{r} - \vec{v}) = \boxed{-6 \vec{i} + 3 \vec{j} + 2 \vec{k}}$

d) $\vec{r} \cdot \vec{v} = 2 \cdot (-4) + 3 \cdot 6 + (-4) \cdot (-2) = -8 + 18 + 8 = 18$

$$r = \sqrt{2^2 + 3^2 + (-4)^2} = 5,38 ; v = \sqrt{(-4)^2 + 6^2 + (-2)^2} = 7,48$$

$$\cos \alpha = \frac{\vec{r} \cdot \vec{v}}{r \cdot v} = \frac{18}{5,38 \cdot 7,48} = 0,447 (9) \quad \alpha = \boxed{63,5^\circ}$$

$$\textcircled{30} \quad r = 10 \quad ; \quad \vec{r} = r \cdot \cos \alpha \cdot \vec{i} + r \cdot \sin \alpha \cdot \vec{j} =$$

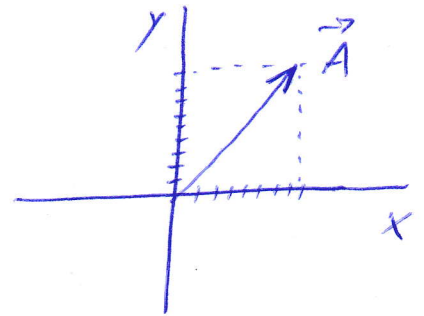
$$= 10 \cdot \sin 30^\circ \cdot \vec{i} + 10 \cdot \cos 30^\circ \cdot \vec{j} = \boxed{8.66 \cdot \vec{i} + 5 \cdot \vec{j}}$$

$$\textcircled{31} \quad A = 12 \quad ; \quad A_x = 8 \quad ; \quad A_x = A \cdot \cos \alpha ;$$

$$\cos \alpha = \frac{A_x}{A} = \frac{8}{12} = 0.667 \Rightarrow \alpha = 48.2^\circ ;$$

$$A_y = A \cdot \sin \alpha = 12 \cdot \sin 48.2^\circ = \boxed{8.95}$$

$$A_x = 8 \cdot \vec{i} + 8.95 \cdot \vec{j}$$



$$\textcircled{32} \quad \vec{PQ} = (5-3, -4-5, 1-5) = (2, -9, -4)$$

$$\vec{u}_{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|} \quad ; \quad |\vec{PQ}| = \sqrt{2^2 + (-9)^2 + (-4)^2} =$$

$$= 10.1$$

$$\vec{u}_{PQ} = \frac{2\vec{i} - 9\vec{j} - 4\vec{k}}{10.1} = \boxed{0.2 \cdot \vec{i} - 0.9 \cdot \vec{j} - 0.4 \cdot \vec{k}}$$

$$\textcircled{33} \quad \vec{D} = \vec{A} - \vec{B} = (5-3, -4-6, 7+4) = (2, -10, 11)$$

$$D = \sqrt{2^2 + 10^2 + 11^2} = 15$$

$$\vec{u}_D = \frac{\vec{D}}{D} = \frac{2\vec{i} - 10\vec{j} + 11\vec{k}}{15} = \boxed{0.133 \cdot \vec{i} - 0.667 \cdot \vec{j} + 0.733 \cdot \vec{k}}$$

$$(34) \quad B=5 ; \quad A=\sqrt{8^2+(-4)^2+(-2)^2} = 9.16$$

$$\vec{u}_A = \frac{\vec{A}}{A} = \frac{8\vec{i} - 4\vec{j} - 2\vec{k}}{9.16} = 0.873.\vec{i} - 0.437.\vec{j} - 0.218.\vec{k}$$

$$\vec{B} = B \cdot \vec{u}_A = 5 \cdot (0.873.\vec{i} - 0.437.\vec{j} - 0.218.\vec{k}) =$$

$$= \boxed{4.36.\vec{i} - 2.18.\vec{j} - 1.09.\vec{k}}$$

$$(35) \quad A=12 ; \quad B=\sqrt{3^2+4^2+(-5)^2} = 7.07$$

$$\vec{u}_B = \frac{\vec{B}}{B} = \frac{3\vec{i} + 4\vec{j} - 5\vec{k}}{7.07} = 0.424.\vec{i} + 0.566.\vec{j} - 0.707.\vec{k}$$

$$\vec{A} = A \cdot \vec{u}_B = 12 \cdot (0.424.\vec{i} + 0.566.\vec{j} - 0.707.\vec{k}) =$$

$$= \boxed{5.09.\vec{i} + 6.79.\vec{j} - 8.48.\vec{k}}$$

$$(36) \quad a) \quad \vec{A} \cdot \vec{B} = 0 \Rightarrow A_x \cdot 2 - 3 \cdot 8 = 0 \Rightarrow A_x \cdot 2 = 24 \Rightarrow$$

$$\Rightarrow A_x = \frac{24}{2} = \boxed{12}$$

$$b) \quad \frac{A_x}{B_x} = \frac{A_y}{B_y} \Rightarrow \frac{A_x}{2} = \frac{-3}{8} \Rightarrow A_x = \frac{2 \cdot (-3)}{8} =$$

$$= -\frac{6}{8} = \boxed{-\frac{3}{4}}$$

(11)

$$(37) \quad a) \quad \vec{A} \cdot \vec{B} = 0 \Rightarrow 4 \cdot 3 - 5 \cdot B_y = 0 \Rightarrow$$

$$\Rightarrow 12 = 5 \cdot B_y \Rightarrow \boxed{B_y = \frac{12}{5}}$$

$$b) \quad \frac{A_x}{B_x} = \frac{A_y}{B_y} \Rightarrow \frac{4}{3} = \frac{-5}{B_y} \Rightarrow B_y = \frac{-5 \cdot 3}{4} = \boxed{-\frac{15}{4}}$$

$$(38) \quad \vec{OA} = (-3+1, 2-3, 1-0) = (-2, -1, 1)$$

$$\vec{OB} = (5+1, -3-3, 2-0) = (6, -6, 2)$$

$$\vec{OA} \cdot \vec{OB} = (-2) \cdot 6 + (-1) \cdot (-6) + 1 \cdot 2 = -12 + 6 + 2 = \boxed{-4}$$

$$(39) \quad \vec{A} \cdot \vec{B} = 3 \cdot (-8) + (-2) \cdot 6 + 4 \cdot (-1) =$$

$$= -24 - 12 - 4 = \boxed{-40}$$

$$A = \sqrt{3^2 + 2^2 + 4^2} = 5\sqrt{38}$$

$$B = \sqrt{8^2 + 6^2 + 1^2} = 10$$

$$\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{A \cdot B} = \frac{-40}{5\sqrt{38} \cdot 10} = -0.743 \Rightarrow \alpha = \boxed{138^\circ}$$

$$\text{proy}_{AB} = \left| \frac{\vec{A} \cdot \vec{B}}{B} \right| = \left| \frac{-40}{10} \right| = \boxed{4}$$

$$\text{proy}_{BA} = \frac{\vec{A} \cdot \vec{B}}{A} = \left| \frac{-40}{5\sqrt{38}} \right| = \boxed{0.743}$$

$$(40) \quad \vec{A} = 6\vec{i} ; \vec{B} = B \cdot \cos \alpha \cdot \vec{i} + B \cdot \sin \alpha \cdot \vec{j} =$$

$$= 5 \cdot \cos 30^\circ \cdot \vec{i} + 5 \cdot \sin 30^\circ \cdot \vec{j} = 4\sqrt{3}\vec{i} + 2\vec{j}$$

$$\vec{C} = -4 \cdot \cos 60^\circ \cdot \vec{i} + 4 \cdot \sin 60^\circ \cdot \vec{j} = -2\vec{i} + 3\sqrt{3}\vec{j}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (6 + 4\sqrt{3} - 2)\vec{i} + (2 + 3\sqrt{3})\vec{j} =$$

$$= \boxed{8\sqrt{3}\vec{i} + 5\sqrt{3}\vec{j}}$$

$$(41) \quad \vec{A} = -8\vec{i} + 6\vec{j} ; \vec{B} = -12\vec{i} - 8\vec{j}$$

$$\vec{S} = \vec{A} + \vec{B} = -20\vec{i} - 2\vec{j} ; \vec{C} = -\vec{S} = \boxed{20\vec{i} + 2\vec{j}}$$

$$(42) \quad a) \quad \vec{P} = 8\vec{i} + 5\vec{j} ; P = \sqrt{8^2 + 5^2} = 9\sqrt{43}$$

$$\vec{u}_P = \frac{\vec{P}}{P} = \frac{8\vec{i} + 5\vec{j}}{9\sqrt{43}} = \boxed{0\sqrt{848}\vec{i} + 0\sqrt{53}\vec{j}}$$

$$b) \quad \vec{B} = B_x \vec{i} + B_y \vec{j} ; B_x = 12 \Rightarrow B \cdot 0\sqrt{848} = 12 \Rightarrow$$

$$\Rightarrow B = \frac{12}{0\sqrt{848}} = 1\sqrt{2} ;$$

$$\vec{B} = B \cdot \vec{u}_P = 1\sqrt{2} \cdot (0\sqrt{848}\vec{i} + 0\sqrt{53}\vec{j}) = \boxed{12\vec{i} + 7\sqrt{53}\vec{j}}$$

$$(43) \quad \vec{P} = 3\vec{i} + 5\vec{j} ; P = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = 5\sqrt{83}$$

$$B = \frac{\vec{P}}{P} = \frac{3\vec{i} + 5\vec{j}}{5\sqrt{83}} = \boxed{0\sqrt{515}\vec{i} + 0\sqrt{858}\vec{j}}$$

$$(44) \quad \vec{A} = 10 \cdot \cos 30^\circ \cdot \vec{i} + 10 \cdot \sin 30^\circ \cdot \vec{j} = 8'66 \cdot \vec{i} + 5 \cdot \vec{j}$$

$$\vec{B} = -15 \cdot \vec{i} ; \quad \vec{C} = -8 \cdot \vec{j}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (8'66 - 15) \cdot \vec{i} + (5 - 8) \cdot \vec{j} = \boxed{-6'34 \cdot \vec{i} - 3 \cdot \vec{j}}$$

$$(45) \quad \vec{A} = 10 \cdot \vec{i} ;$$

$$\vec{B} = -15 \cdot \cos 30^\circ \cdot \vec{i} + 15 \cdot \sin 30^\circ \cdot \vec{j} = -13 \cdot \vec{i} + 7'5 \cdot \vec{j}$$

$$\vec{C} = -22 \cdot \cos 60^\circ \cdot \vec{i} - 22 \cdot \sin 60^\circ \cdot \vec{j} = -11 \cdot \vec{i} - 19'1 \cdot \vec{j}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (10 - 13 - 11) \cdot \vec{i} + (7'5 - 19'1) \cdot \vec{j} = \boxed{-14 \cdot \vec{i} - 11'6 \cdot \vec{j}}$$

$$(46) \quad a) \quad A = \sqrt{3^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \boxed{7}$$

$$b) \quad \cos \alpha = \frac{A_x}{A} = \frac{3}{7} = 0'429 \Rightarrow \alpha = \boxed{64'6^\circ}$$

$$c) \quad \cos \beta = \frac{A_y}{A} = \frac{-2}{7} = -0'286 \Rightarrow \beta = \boxed{107^\circ}$$

$$d) \quad \cos \gamma = \frac{A_z}{A} = \frac{6}{7} = 0'857 \Rightarrow \gamma = \boxed{31^\circ}$$

$$e) \quad \vec{u}_A = \frac{\vec{A}}{A} = \frac{3\vec{i} - 2\vec{j} + 6\vec{k}}{7} = \boxed{0'429 \cdot \vec{i} - 0'286 \cdot \vec{j} + 0'857 \cdot \vec{k}}$$

47) a) $\vec{A} + \vec{B} = (3-4, -2+4, 6+8) = \boxed{(-1, 2, 14)}$

b) $\vec{B} - \vec{A} = (-4-3, 4+2, 8-6) = \boxed{(-7, 6, 2)}$

c) $2\vec{A} - 3\vec{B} = 2 \cdot (3\vec{i} - 2\vec{j} + 6\vec{k}) - 3 \cdot (-4\vec{i} + 4\vec{j} + 8\vec{k}) =$
 $= 6\vec{i} - 4\vec{j} + 12\vec{k} + 12\vec{i} - 12\vec{j} - 24\vec{k} =$
 $= (6+12) \cdot \vec{i} + (-4-12) \cdot \vec{j} + (12-24) \cdot \vec{k} = \boxed{18\vec{i} - 16\vec{j} - 12\vec{k}}$

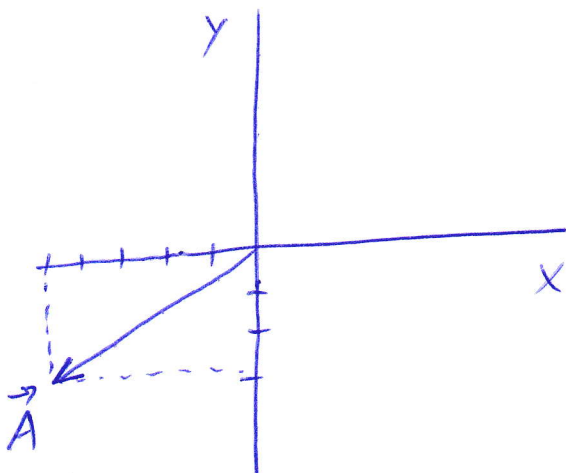
d) $|2\vec{A} - 3\vec{B}| = \sqrt{18^2 + 16^2 + 12^2} = \boxed{26'9}$

e) $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 6 \\ -4 & 4 & 8 \end{vmatrix} =$

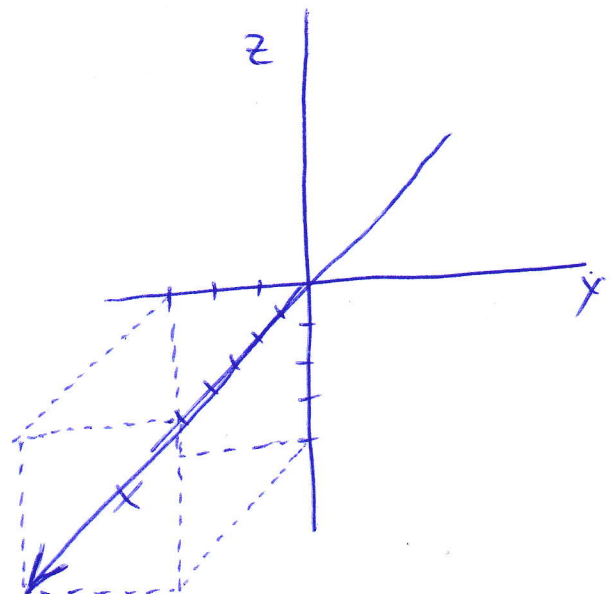
$= -16\vec{i} + 12\vec{k} - 24\vec{j} - 8\vec{k} - 24\vec{i} - 24\vec{j} = -40\vec{i} - 48\vec{j} + 4\vec{k}$

$|\vec{A} \times \vec{B}| = \sqrt{40^2 + 48^2 + 4^2} = \boxed{62'6}$

48) a)



b)



$$(49) \quad A = 46; \quad \alpha = 90 - \beta = 90 - 33 = 57^\circ$$

$$\begin{aligned} \vec{A} &= A \cdot \cos \alpha \cdot \vec{i} + A \cdot \sin \alpha \cdot \vec{j} = 46 \cdot \cos 57^\circ \cdot \vec{i} + 46 \cdot \sin 57^\circ \cdot \vec{j} = \\ &= \boxed{251 \cdot \vec{i} + 386 \cdot \vec{j}} \end{aligned}$$

$$\begin{aligned} (50) \quad 2\vec{A} - \vec{B} &= 2 \cdot (\vec{i} + \vec{j}) - (m\vec{i} + 3\vec{j}) = \\ &= 2\vec{i} + 2\vec{j} - m\vec{i} - 3\vec{j} = (2-m) \cdot \vec{i} - \vec{j} \end{aligned}$$

$$|2\vec{A} - \vec{B}| = \sqrt{(2-m)^2 + (-1)^2} = 1 \Rightarrow$$

$$(2-m)^2 + 1^2 = 1 \Rightarrow 2-m=0 \Rightarrow \boxed{m=2}$$

$$\begin{aligned} (51) \quad \vec{A} - 3\vec{B} &= 3\vec{i} - 7\vec{j} - 2\vec{k} - 3 \cdot (5\vec{i} + 4\vec{j} + 2\vec{k}) = \\ &= 3\vec{i} - 7\vec{j} - 2\vec{k} - 15\vec{i} - 12\vec{j} - 6\vec{k} = \\ &= -12\vec{i} - 19\vec{j} - 8\vec{k} \end{aligned}$$

$$\begin{aligned} 2\vec{A} - 4\vec{B} &= 2 \cdot (3\vec{i} - 7\vec{j} - 2\vec{k}) - 4 \cdot (5\vec{i} + 4\vec{j} + 2\vec{k}) = \\ &= 6\vec{i} - 14\vec{j} - 4\vec{k} - 20\vec{i} - 16\vec{j} - 8\vec{k} = \\ &= -14\vec{i} - 30\vec{j} - 12\vec{k} \end{aligned}$$

$$\begin{aligned} (\vec{A} - 3\vec{B}) \cdot (2\vec{A} - 4\vec{B}) &= (-12) \cdot (-14) + (-19) \cdot (-30) + (-8) \cdot (-12) = \\ &= \boxed{834} \end{aligned}$$

52) a)

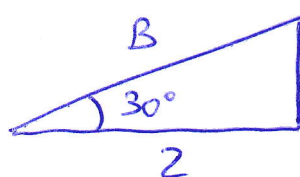
$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 6 \\ -4 & 4 & 8 \end{vmatrix} = -16\vec{i} + 12\vec{k} - 24\vec{j} - 8\vec{k} - 24\vec{i} - 24\vec{j} = \boxed{-40\vec{i} - 48\vec{j} + 4\vec{k}}$$

b)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -4 & 7 \\ 3 & 6 & -4 \end{vmatrix} = 16\vec{i} + 30\vec{k} + 21\vec{j} + 12\vec{k} - 42\vec{i} + 20\vec{j} = \boxed{-26\vec{i} + 41\vec{j} + 42\vec{k}}$$

53) a) $\vec{A} = -A_x \cdot \vec{i} + A_y \cdot \vec{j} = \boxed{-2 \cdot \vec{i} + 2 \cdot \vec{j}}$

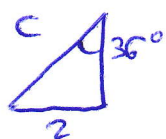
b) $\vec{B} = B \cdot \cos \alpha \cdot \vec{i} + B \cdot \sin \alpha \cdot \vec{j}$;



$$\cos 30^\circ = \frac{2}{B} \Rightarrow B = \frac{2}{\cos 30^\circ} = 2'31$$

$$\vec{B} = 2'31 \cdot \cos 30^\circ \cdot \vec{i} + 2'31 \cdot \sin 30^\circ \cdot \vec{j} = \boxed{2 \cdot \vec{i} + 1'16 \cdot \vec{j}}$$

c) $\frac{360^\circ}{5} = 72^\circ$; $\frac{72}{2} = 36^\circ$



$$\sin 36^\circ = \frac{2}{C} \Rightarrow C = \frac{2}{\sin 36^\circ} = 3'4$$

$$\vec{C} = -C \cdot \cos \alpha \cdot \vec{i} - C \cdot \sin \alpha \cdot \vec{j} = -3'4 \cdot \cos 54^\circ \cdot \vec{i} - 3'4 \cdot \sin 54^\circ \cdot \vec{j} = \boxed{-2 \cdot \vec{i} - 2'75 \cdot \vec{j}}$$

d) $\vec{D} = 6 \cdot \cos 60^\circ \cdot \vec{i} - 6 \cdot \sin 60^\circ \cdot \vec{j} = \boxed{3 \cdot \vec{i} - 5'2 \cdot \vec{j}}$

(17)

PROBLEMAS RESUELTOS DE CINEMÁTICA

$$\textcircled{1} \text{ a) } y' = 6 \cdot 4 \cdot x^3 + 8 \cdot 3 \cdot x^2 - 3 \cdot 2 \cdot x + 7 =$$

$$= \boxed{24 \cdot x^3 + 24 \cdot x^2 - 6 \cdot x + 7}$$

$$\text{b) } y = \sqrt{f(x)} \quad ; \quad y' = \frac{f'(x)}{2 \cdot \sqrt{f(x)}}$$

$$y' = \frac{-6x + 5}{2 \cdot \sqrt{-3x^2 + 5x + 7}}$$

$$\text{c) } y = a \cdot \text{sen}(k \cdot x) \quad ; \quad y' = a \cdot k \cdot \text{cos}(k \cdot x)$$

$$y' = 8 \cdot 3 \cdot \text{cos } 3x = \boxed{24 \cdot \text{cos } 3x}$$

$$\textcircled{2} \text{ a) } \boxed{y' = -15 \cdot x^4 + 18 \cdot x^2 - 4 \cdot x + 1}$$

$$\text{b) } y = \sqrt{f(x)} \quad ; \quad y' = \frac{f'(x)}{2 \cdot \sqrt{f(x)}}$$

$$y' = \frac{12x^3 + 10x}{2 \cdot \sqrt{3x^4 + 5x^2 + 7}} = \boxed{\frac{6x^3 + 5x}{\sqrt{3x^4 + 5x^2 + 7}}}$$

$$\text{c) } y = a \cdot \text{cos}(k \cdot x) \quad ; \quad y' = -a \cdot k \cdot \text{sen}(k \cdot x)$$

$$y' = -7 \cdot 2 \cdot \text{sen}(2x) = \boxed{-14 \cdot \text{sen } 2x}$$

③ * De 0 a 20 s: $MAUR$:

$$a = \frac{\Delta v}{\Delta t} = \frac{10-4}{20-0} = \frac{6}{20} = \frac{3}{10} = \boxed{0,3 \frac{m}{s^2}}$$

* De 20 a 40 s: MAU ; $\boxed{a=0}$

④ * De 0 a 20 s: $MAUA$

$$a = \frac{\Delta v}{\Delta t} = \frac{10-4}{20-0} = \frac{6}{20} = \frac{3}{10} = \boxed{0,3 \frac{m}{s^2}}$$

* De 20 a 40 s: $MAU \Rightarrow \boxed{a=0}$

* De 40 a 60 s: $MAUR$

$$a = \frac{\Delta v}{\Delta t} = \frac{10-0}{60-40} = \frac{10}{20} = \boxed{0,5 \frac{m}{s^2}}$$

⑤ * De 0 a 30 s: $MAUR$

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{12-4}{30-0} = \frac{8}{30} = 0,267 \frac{m}{s^2}$$

$$e_1 = v_0 \cdot t - \frac{1}{2} \cdot a_1 \cdot t^2 = 12 \cdot 30 - \frac{1}{2} \cdot 0,267 \cdot 30^2 = 240 \text{ m}$$

* De 30 a 60 s: MAU

$$e_2 = v \cdot t = 4 \cdot 30 = 120 \text{ m}$$

* De 60 a 80 s: $MAUA$

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{8-4}{80-60} = \frac{4}{20} = 0,2 \frac{m}{s^2}$$

$$e_3 = v_0 \cdot t + \frac{1}{2} \cdot a_3 \cdot t^2 = 4 \cdot 20 + \frac{1}{2} \cdot 0,2 \cdot 20^2 = 120 \text{ m}$$

$$(19) \quad e_T = 240 + 120 + 120 = \boxed{480 \text{ m}}$$

$$\begin{aligned}
 \textcircled{6} \quad v_1 &= 80 \frac{\text{km}}{\text{h}} \\
 t_1 &= 2 \text{ h} \\
 v_2 &= 120 \frac{\text{km}}{\text{h}} \\
 t_2 &= 3 \text{ h} \\
 \hat{v}_m?
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 v_m &= \frac{e_T}{t_T} = \frac{e_1 + e_2}{t_1 + t_2} = \frac{v_1 \cdot t_1 + v_2 \cdot t_2}{t_1 + t_2} = \\
 &= \frac{80 \cdot 2 + 120 \cdot 3}{2 + 3} = \boxed{104 \frac{\text{km}}{\text{h}}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{7} \quad v &= 100 \frac{\text{km}}{\text{h}} = 27'8 \frac{\text{m}}{\text{s}} \\
 t &= 1 \text{ min} \\
 v &= 120 \frac{\text{km}}{\text{h}} = 33'3 \frac{\text{m}}{\text{s}} \\
 \hat{a}?
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 a &= \frac{\Delta v}{\Delta t} = \frac{33'3 - 27'8}{60} = \\
 &= \boxed{0'0917 \frac{\text{m}}{\text{s}^2}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad t_1 &= 3 \text{ h } 29 \text{ min } 52 \text{ s} = 3'498 \text{ h} \\
 t_2 &= 1 \text{ h } 42 \text{ min } 18 \text{ s} = 1'705 \text{ h} \\
 e_1 &= 42'195 \text{ km} \\
 e_2 &= 21'097 \text{ km} \\
 \hat{v}, \text{ ritmo?}
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 v_1 &= \frac{e_1}{t_1} = \frac{42'195}{3'498} = \\
 &= \boxed{12'1 \frac{\text{km}}{\text{h}}} \\
 v_2 &= \frac{e_2}{t_2} = \frac{21'097}{1'705} = \\
 &= \boxed{12'4 \frac{\text{km}}{\text{h}}}
 \end{aligned}$$

$$\text{Ritmo}_1 = \frac{1 \text{ K}}{12'1 \text{ km}} \cdot \frac{60 \text{ min}}{1 \text{ K}} = \boxed{4'96 \frac{\text{min}}{\text{km}}}$$

$$\text{Ritmo}_2 = \frac{1 \text{ K}}{12'4 \text{ km}} \cdot \frac{60 \text{ min}}{1 \text{ K}} = \boxed{4'84 \frac{\text{min}}{\text{km}}}$$

$$\textcircled{9} \text{ a) } s(2) = 5 \cdot 2^3 - 3 \cdot 2^2 + 5 = 5 \cdot 8 - 12 + 5 = 45 - 12 = \boxed{33 \text{ m}}$$

$$s(4) = 5 \cdot 4^3 - 3 \cdot 4^2 + 5 = \boxed{277 \text{ m}}$$

$$\text{b) } \Delta s = s(4) - s(2) = 277 - 33 = \boxed{244 \text{ m}}$$

$$\text{c) } v = \frac{ds}{dt} = \boxed{15 \cdot t^2 - 6 \cdot t}$$

$$\text{d) } v(3) = 15 \cdot 3^2 - 6 \cdot 3 = 15 \cdot 9 - 18 = 135 - 18 = \boxed{117 \frac{\text{m}}{\text{s}}}$$

$$\text{e) } a = \frac{dv}{dt} = \boxed{30 \cdot t - 6}$$

$$\text{f) } a(3) = 30 \cdot 3 - 6 = 90 - 6 = \boxed{84 \frac{\text{m}}{\text{s}^2}}$$

$$\text{g) } \vec{r} = s \cdot \vec{l} = \boxed{(15 \cdot t^3 - 3 \cdot t^2 + 5) \cdot \vec{l}}$$

$$\text{h) } v_m = \frac{\Delta s}{\Delta t} = \frac{244}{2} = \boxed{122 \frac{\text{m}}{\text{s}}}$$

$$\text{i) } a_m = \frac{\Delta v}{\Delta t} = \frac{15 \cdot 4^2 - 6 \cdot 4 - 15 \cdot 2^2 + 6 \cdot 2}{4 - 2} =$$

$$= \frac{168}{2} = \boxed{84 \frac{\text{m}}{\text{s}^2}}$$

10) $e_1 = 18 \text{ km}$

$e_2 = 24 \text{ km}$

$t_1 = 40 \text{ min}$

$t_2 = 20 \text{ min}$

Find V_{m1}, V_{m2}, V_{mT} ?

$$V_{m1} = \frac{e_1}{t_1} = \frac{18}{\frac{40}{60}} = \frac{18 \cdot 60}{40} = \boxed{27 \frac{\text{km}}{\text{h}}}$$

$$V_{m2} = \frac{e_2}{t_2} = \frac{24}{\frac{20}{60}} = \frac{24 \cdot 60}{20} = \boxed{72 \frac{\text{km}}{\text{h}}}$$

$$V_{mT} = \frac{e_T}{t_T} = \frac{18 + 24}{\frac{40}{60} + \frac{20}{60}} = \boxed{42 \frac{\text{km}}{\text{h}}}$$

11) $V_2 = 10 \cdot V_1$

$t_1 = 7 \cdot t_2$

$V_1 = 10 \frac{\text{km}}{\text{h}}$

Find V_m ?

$$\begin{aligned} V_m &= \frac{e_T}{t_T} = \frac{e_1 + e_2}{t_1 + t_2} = \frac{V_1 \cdot t_1 + V_2 \cdot t_2}{t_1 + t_2} = \\ &= \frac{10 \cdot 7 \cdot t_2 + 100 \cdot t_2}{7 \cdot t_2 + t_2} = \frac{170 \cdot t_2}{8 \cdot t_2} = \\ &= \boxed{21.25 \frac{\text{km}}{\text{h}}} \end{aligned}$$

12) $V_1 = 20 \frac{\text{km}}{\text{h}}$

$V_2 = 5 \frac{\text{km}}{\text{h}}$

Find V_m ?

$$\begin{aligned} V_m &= \frac{e_T}{t_T} = \frac{e_1 + e_2}{t_1 + t_2} = \frac{e + e}{\frac{e}{V_1} + \frac{e}{V_2}} = \\ &= \frac{2 \cdot \cancel{e}}{\cancel{e} \cdot \left(\frac{1}{V_1} + \frac{1}{V_2} \right)} = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}} = \end{aligned}$$

$$= \frac{2}{\frac{1}{20} + \frac{1}{5}} = \boxed{8 \frac{\text{km}}{\text{h}}}$$

$$13) a) \vec{v} = \frac{d\vec{r}}{dt} = \boxed{3 \cdot t^2 \cdot \vec{i} + 4 \cdot t \cdot \vec{j} - 5 \cdot \vec{k}}$$

$$b) \vec{a} = \frac{d\vec{v}}{dt} = \boxed{6 \cdot t \cdot \vec{i} + 4 \cdot \vec{j}}$$

$$c) \vec{r}(1) = \vec{i} - \vec{j} - 5 \cdot \vec{k} ; \vec{r}(3) = 3^3 \cdot \vec{i} + (2 \cdot 3^2 - 3) \cdot \vec{j} - 5 \cdot 3 \cdot \vec{k} = 27 \cdot \vec{i} + 15 \cdot \vec{j} - 15 \cdot \vec{k}$$

$$\Delta \vec{r} = \vec{r}(3) - \vec{r}(1) = \boxed{26 \cdot \vec{i} + 16 \cdot \vec{j} - 10 \cdot \vec{k}}$$

$$d) \vec{v}_m = \frac{\Delta \vec{r}}{\Delta t} = \frac{26 \cdot \vec{i} + 16 \cdot \vec{j} - 10 \cdot \vec{k}}{3-1} = \boxed{13 \cdot \vec{i} + 8 \cdot \vec{j} - 5 \cdot \vec{k}}$$

$$e) \vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{3 \cdot 3^2 \cdot \vec{i} + 4 \cdot 3 \cdot \vec{j} - 5 \cdot \vec{k} - 3 \cdot \vec{i} - 4 \cdot \vec{j} + 5 \cdot \vec{k}}{3-1} =$$

$$= \frac{24 \cdot \vec{i} + 8 \cdot \vec{j}}{2} = \boxed{12 \cdot \vec{i} + 4 \cdot \vec{j}}$$

$$f) \vec{v}(4) = 3 \cdot 4^2 \cdot \vec{i} + 4 \cdot 4 \cdot \vec{j} - 5 \cdot \vec{k} = \boxed{48 \cdot \vec{i} + 16 \cdot \vec{j} - 5 \cdot \vec{k}}$$

$$g) v(4) = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{48^2 + 16^2 + 5^2} = \boxed{50.8 \frac{m}{s}}$$

$$h) \vec{a}(5) = 6 \cdot 5 \cdot \vec{i} + 4 \cdot \vec{j} = \boxed{30 \cdot \vec{i} + 4 \cdot \vec{j}}$$

$$i) a(5) = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 4^2} = \boxed{30.3 \frac{m}{s^2}}$$

$$(14) \quad \vec{r} = 5t^3 \cdot \vec{i} + 6t^2 \cdot \vec{j}$$

$$a_t, a_n?$$

$$R?$$

$$t = 3 \text{ s}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 15t^2 \cdot \vec{i} + 12t \cdot \vec{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30t \cdot \vec{i} + 12 \cdot \vec{j}$$

$$\vec{a}(3) = 30 \cdot 3 \cdot \vec{i} + 12 \cdot \vec{j} = 90 \cdot \vec{i} + 12 \cdot \vec{j}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{90^2 + 12^2} = 90.8 \frac{\text{m}}{\text{s}^2}$$

$$v(t) = \sqrt{v_x^2 + v_y^2} = \sqrt{(15t^2)^2 + (12t)^2} = \sqrt{225t^4 + 144t^2}$$

$$a_t(3) = \frac{dv(t)}{dt} = \frac{4 \cdot 225 \cdot t^3 + 2 \cdot 144 \cdot t}{2 \cdot \sqrt{225t^4 + 144t^2}} =$$

$$= \frac{4 \cdot 225 \cdot 3^3 + 2 \cdot 144 \cdot 3}{2 \cdot \sqrt{225 \cdot 3^4 + 144 \cdot 3^2}} = \frac{2'52 \cdot 10^4}{279} = \boxed{90.3 \frac{\text{m}}{\text{s}^2}}$$

$$v(3) = \sqrt{225 \cdot 3^4 + 144 \cdot 3^2} = 140 \frac{\text{m}}{\text{s}}$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{90.8^2 - 90.3^2} = \boxed{9.52 \frac{\text{m}}{\text{s}^2}}$$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_n} = \frac{140^2}{9.52} = \boxed{2059 \text{ m}}$$

15) $x = 1 + \sin(\pi \cdot t)$

$y = t - \cos(\pi \cdot t)$

$\vec{v}, \vec{a}?$

$a_t, a_n? \quad t = 2 \text{ s}$

$R? \quad t = 2 \text{ s}$

a) $\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} =$

$= (1 + \sin \pi t) \cdot \vec{i} + (t - \cos \pi t) \cdot \vec{j}$

$\vec{v} = \pi \cdot \cos \pi t \cdot \vec{i} + (1 + \pi \cdot \sin \pi t) \cdot \vec{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = [-\pi^2 \cdot \sin \pi t \cdot \vec{i} + \pi^2 \cdot \cos \pi t \cdot \vec{j}]$

b) $\vec{a}(2) = -\pi^2 \cdot \sin 2\pi \cdot \vec{i} + \pi^2 \cdot \cos 2\pi \cdot \vec{j} = \pi^2 \cdot \vec{j} \Rightarrow$

$\Rightarrow a = \pi^2 = 9.87 \frac{\text{m}}{\text{s}^2}$

$v(t) = \sqrt{v_x^2 + v_y^2} = \sqrt{\pi^2 \cdot \cos^2 \pi t + (1 + \pi \cdot \sin \pi t)^2} =$

$= \sqrt{\pi^2 \cdot \cos^2 \pi t + 1 + \pi^2 \cdot \sin^2 \pi t + 2\pi \cdot \sin \pi t} =$

$= \sqrt{\pi^2 + 1 + 2\pi \cdot \sin \pi t}$

$a_t = \frac{dv(t)}{dt} = \frac{\cancel{2} \cdot \pi^2 \cdot \cos \pi t}{\cancel{2} \cdot \sqrt{\pi^2 + 1 + 2\pi \cdot \sin \pi t}} = \frac{\pi^2 \cdot \cos 2\pi}{\sqrt{\pi^2 + 1 + 2\pi \cdot 0}} =$

$= \frac{\pi^2}{\sqrt{\pi^2 + 1}} = \boxed{2.99 \frac{\text{m}}{\text{s}^2}}$

$a_n = \sqrt{a^2 - a_t^2} = \sqrt{9.87^2 - 2.99^2} = \boxed{9.41 \frac{\text{m}}{\text{s}^2}}$

c) $a_n = \frac{v^2}{R}; \quad v(2) = \sqrt{\pi^2 + 1 + 2\pi \cdot 0} = \sqrt{\pi^2 + 1} = 3.3 \frac{\text{m}}{\text{s}}$

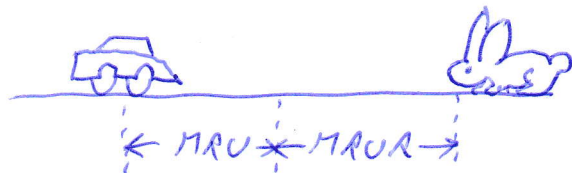
$R = \frac{v^2}{a_n} = \frac{3.3^2}{9.41} = \boxed{1.16 \text{ m}}$

$$(16) \quad v_0 = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$$

$$d = 130 \text{ m}$$

$$t_R = 1 \text{ s}$$

$$a = 3 \frac{\text{m}}{\text{s}^2}$$



$$e_T = e_1 + e_2$$

$$e_1 = v \cdot t = 27.8 \cdot 1 = 27.8 \text{ m}$$

$$v^2 = v_0^2 - 2 \cdot a \cdot e_2 \Rightarrow e_2 = \frac{v_0^2 - v^2}{2 \cdot a} = \frac{27.8^2 - 0^2}{2 \cdot 3} = 129 \text{ m}$$

$$e_T = e_1 + e_2 = 27.8 + 129 = 157 \text{ m} > d = 130 \text{ m} \Rightarrow$$

$\Rightarrow$ Lo pillará

$$(17) \quad R = 0.08 \text{ m}$$

$$\omega = 200 \text{ rpm}$$

$$t = 6 \text{ s}$$

$$t = 30 \text{ s}$$

$$t = 70 \text{ s}$$

$$a) \dot{\varphi}?$$

$$b) \dot{N}?$$

$$c) \dot{e}?$$

$$\omega = 200 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 20.9 \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = \frac{\Delta \omega}{\Delta t} = \frac{20.9 - 0}{6} = 3.48 \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_3 = \frac{\Delta \omega}{\Delta t} = \frac{20.9 - 0}{70} = 0.299 \frac{\text{rad}}{\text{s}^2}$$

$$\varphi_1 = \omega_0 \cdot t + \frac{1}{2} \cdot \alpha_1 \cdot t^2 = 0 + \frac{1}{2} \cdot 3.48 \cdot 6^2 = 62.6 \text{ rad}$$

$$\varphi_2 = \omega \cdot t = 20.9 \cdot 30 = 627 \text{ rad}$$

$$\varphi_3 = \omega_0 \cdot t - \frac{1}{2} \cdot \alpha \cdot t^2 = 20.9 \cdot 70 - \frac{1}{2} \cdot 0.299 \cdot 70^2 = 731 \text{ rad}$$

$$\varphi_T = \varphi_1 + \varphi_2 + \varphi_3 = 62.6 + 627 + 731 = \boxed{1420 \text{ rad}}$$

$$N = \frac{\varphi}{2\pi} = \frac{1420}{2\pi} = \boxed{226 \text{ vueltas}}$$

$$e = \varphi \cdot R = 1420 \cdot 0.08 = \boxed{114 \text{ m}}$$

(26)

18) ω, T, f ?

* Segundos: $T = 60 \text{ s}$; $f = \frac{1}{T} = \frac{1}{60} = 0.0167 \text{ Hz}$

$$\omega = \frac{\varphi}{t} = \frac{2\pi}{T} = \frac{2 \cdot \pi}{60} = 0.105 \frac{\text{rad}}{\text{s}}$$

* Minutos: $T = 1 \text{ h} = 3600 \text{ s}$; $f = \frac{1}{T} = \frac{1}{3600} = 2.78 \cdot 10^{-4} \text{ Hz}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3600} = 1.75 \cdot 10^{-3} \frac{\text{rad}}{\text{s}}$$

* Horas: $T = 12 \text{ h} = 12 \cdot 3600 = 4.32 \cdot 10^4 \text{ s}$

$$f = \frac{1}{T} = \frac{1}{4.32 \cdot 10^4} = 2.31 \cdot 10^{-5} \text{ Hz}$$

$$\omega = \frac{2\pi}{T} = \frac{2 \cdot \pi}{4.32 \cdot 10^4} = 1.45 \cdot 10^{-4} \frac{\text{rad}}{\text{s}}$$

19) ω, v, a_c ?

$$R = 6370 \text{ km}$$

$$\omega = \frac{2\pi}{T} = \frac{2 \cdot \pi}{24 \cdot 3600} = 7.27 \cdot 10^{-5} \frac{\text{rad}}{\text{s}}$$

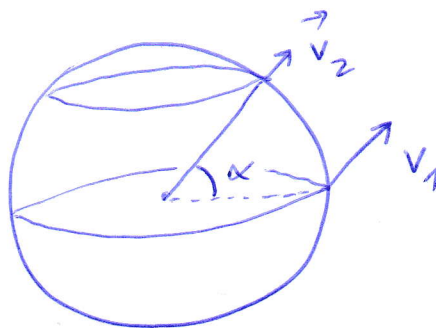
$$v = \omega \cdot R = 7.27 \cdot 10^{-5} \cdot 6.37 \cdot 10^6 = 463 \frac{\text{m}}{\text{s}}$$

$$a_c = \frac{v^2}{R} = \frac{463^2}{6.37 \cdot 10^6} = 0.0337 \frac{\text{m}}{\text{s}^2}$$

(20) $\vec{v}, a_n?$

$\alpha = 60^\circ$

$R = 6370 \text{ km}$



$$\omega = \frac{2\pi}{T} = \frac{2 \cdot \pi}{24 \cdot 3600} = 727 \cdot 10^{-5} \frac{\text{rad}}{\text{s}}$$

* En el ecuador: $v_1 = \omega \cdot R = 727 \cdot 10^{-5} \cdot 637 \cdot 10^6 = 463 \frac{\text{m}}{\text{s}}$

* A 60° norte:

$$v_2 = \omega \cdot R \cdot \cos \alpha = 727 \cdot 10^{-5} \cdot 637 \cdot 10^6 \cdot \cos 60^\circ = 232 \frac{\text{m}}{\text{s}} = \boxed{834 \frac{\text{km}}{\text{h}}}$$

$$a_n = \frac{v_2^2}{R_2} = \frac{v_2^2}{R \cdot \cos \alpha} = \frac{232^2}{637 \cdot 10^6 \cdot \cos 60^\circ} = \boxed{0.0169 \frac{\text{m}}{\text{s}^2}}$$

(21) $v = 10 \frac{\text{km}}{\text{h}} = 2.78 \frac{\text{m}}{\text{s}}$

$v = 30 \frac{\text{km}}{\text{h}} = 8.33 \frac{\text{m}}{\text{s}}$

$d = 25 \text{ m}$

* Velocidad del hombre a 5 s:

$$v = v_0 + a \cdot t = 2.78 + 1 \cdot 5 = 7.78 \frac{\text{m}}{\text{s}}$$

* Espacio recorrido por el hombre a los 5 s:

$$e = v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 = 2.78 \cdot 5 + \frac{1}{2} \cdot 1 \cdot 5^2 = 26.4 \text{ m}$$

* Espacio recorrido por el perro a los 5 s:

$$e = v \cdot t = 8.33 \cdot 5 = 41.7 \text{ m}$$

* Distancia que los separa a los 5 s:

$$d = 25 + 26.4 - 41.7 = 9.7 \text{ m}$$

P: $9.7 + x = 8.33 \cdot t$; H: $x = 7.78 \cdot t \Rightarrow 9.7 + 7.78 \cdot t = 8.33 \cdot t \Rightarrow$

$$\Rightarrow t = 17.6 \text{ s} \Rightarrow t_T = 5 + 17.6 = \boxed{22.6 \text{ s}}$$

$x = 7.78 \cdot t = 7.78 \cdot 17.6 = 137 \text{ m}$; H: $26.4 + 137 = \boxed{163 \text{ m}}$

(28)

P: $163 + 25 = \boxed{188 \text{ m}}$

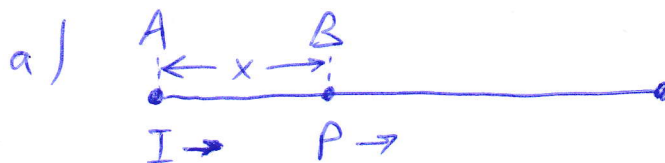
(22) $v_I = 25 \frac{\text{km}}{\text{h}}$

$v_P = 15 \frac{\text{km}}{\text{h}}$

$x?$

$d = 2 \text{ km}$

$t?$



$e = v \cdot t$

Indurdaín: $2 = 25 \cdot t$

Petico: $2 - x = 15 \cdot t$

$t = \frac{2}{25} = 0'08 \text{ h} \Rightarrow x = 2 - 15 \cdot t = 2 - 15 \cdot 0'08 = 0'8 \text{ km} = \boxed{800 \text{ m}}$

b) Indurdaín: $2 = 25 \cdot t_1$ $t_1 = \frac{2}{25} = 0'08 \text{ h}$
 Petico: $2 = 15 \cdot t_2$ $t_2 = \frac{2}{15} = 0'133 \text{ h}$

$\Delta t = t_2 - t_1 = 0'133 - 0'08 = 0'053 \text{ h} = \boxed{191 \text{ s}}$

(23) $d_A = 50 \text{ m}$

$v_A = 25 \frac{\text{km}}{\text{h}} = 6'94 \frac{\text{m}}{\text{s}}$

$d_B = 56 \text{ m}$

$v_B = 29 \frac{\text{km}}{\text{h}} = 8'06 \frac{\text{m}}{\text{s}}$

$t = \frac{d}{v}$

$t_A = \frac{d_A}{v_A} = \frac{50}{6'94} = 7'21 \text{ s}$

$t_B = \frac{d_B}{v_B} = \frac{56}{8'06} = 6'95 \text{ s}$

Ganará el B porque tardará menos.

$$\begin{array}{l|l} \textcircled{24} \quad v_0 = 4 \frac{\text{m}}{\text{s}} & e = v_0 \cdot t + \frac{1}{2} \cdot g \cdot t^2 \\ h = 50 \text{ m} & 50 = 4 \cdot t + \frac{1}{2} \cdot 10 \cdot t^2 \Rightarrow \\ \text{¿ } t, v? & \Rightarrow 5 \cdot t^2 + 4 \cdot t - 50 = 0 \end{array}$$

$$t = \frac{-4 \pm \sqrt{4^2 + 4 \cdot 5 \cdot 50}}{2 \cdot 5} = \boxed{2.79 \text{ s}}$$

$$v = v_0 + g \cdot t = 4 + 10 \cdot 2.79 = \boxed{31.9 \frac{\text{m}}{\text{s}}}$$

$$\begin{array}{l|l} \textcircled{25} \quad t = 1.5 \text{ s} & * \text{ sin tener en cuenta la velocidad del} \\ \text{¿ } h? & \text{sonido:} \\ g = 9.8 \frac{\text{m}}{\text{s}^2} & e = \frac{1}{2} \cdot g \cdot t^2 = \frac{1}{2} \cdot 9.8 \cdot 1.5^2 = \boxed{11.03 \text{ m}} \\ v_{\text{sonido}} = 340 \frac{\text{m}}{\text{s}} & \end{array}$$

* Teniéndola en cuenta:

$$\left. \begin{array}{l} t_T = t_1 + t_2 = 1.5 \text{ s} \\ e = \frac{1}{2} \cdot g \cdot t_1^2 = 4.9 \cdot t_1^2 \\ e = v \cdot t_2 = 340 \cdot t_2 \end{array} \right\} \begin{array}{l} 4.9 \cdot t_1^2 = 340 \cdot t_2 \Rightarrow t_2 = 0.0144 \cdot t_1^2 \\ t_1 + 0.0144 \cdot t_1^2 = 1.5 \Rightarrow \\ \Rightarrow 0.0144 \cdot t_1^2 + t_1 - 1.5 = 0 \end{array}$$

$$t_1 = \frac{-1 \pm \sqrt{1 + 4 \cdot 0.0144 \cdot 1.5}}{2 \cdot 0.0144} = 1.47 \text{ s}$$

$$e = 4.9 \cdot t_1^2 = 4.9 \cdot 1.47^2 = \boxed{10.59 \text{ m}}$$

(26) $v_0 = 0$

$$v = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$$

$$e = 52 \text{ m}$$

$$t = 30 \text{ s}$$

$$t = 10 \text{ s}$$

$$e_T?$$

$$e_T = e_1 + e_2 + e_3$$

$$e_1 = 52 \text{ m}$$

$$e_2 = 27.8 \cdot 30 = 834 \text{ m}$$

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{27.8 - 0}{10 - 0} = 2.78 \frac{\text{m}}{\text{s}^2}$$

$$e_3 = v_0 \cdot t - \frac{1}{2} \cdot a \cdot t^2 =$$

$$= 27.8 \cdot 10 - \frac{1}{2} \cdot 2.78 \cdot 10^2 = 139 \text{ m}$$

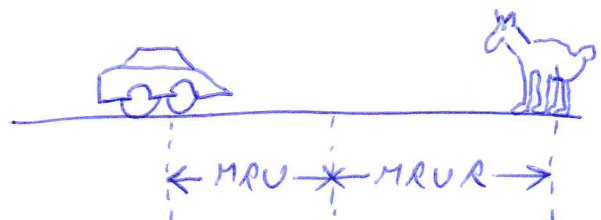
$$e_T = e_1 + e_2 + e_3 = 52 + 834 + 139 = \boxed{1025 \text{ m}}$$

(27) $v_0 = 80 \frac{\text{km}}{\text{h}} = 22.2 \frac{\text{m}}{\text{s}}$

$$d = 200 \text{ m}$$

$$t_R = 0.9 \text{ s}$$

$$a?$$



$$e_T = e_1 + e_2$$

$$e_1 = v \cdot t_R = 22.2 \cdot 0.9 = 20 \text{ m}$$

$$e_2 = 200 - 20 = 180 \text{ m}$$

$$v^2 = v_0^2 - 2 \cdot a \cdot e_2 \Rightarrow a = \frac{v_0^2 - v^2}{2 \cdot e_2} =$$

$$= \frac{22.2^2 - 0^2}{2 \cdot 180} = \boxed{1.37 \frac{\text{m}}{\text{s}^2}}$$

(28) $v_0 = 400 \frac{\text{km}}{\text{h}} = 111 \frac{\text{m}}{\text{s}}$

$h = 2 \text{ km}$

a) $\vec{r}(t)?$
 b) $\vec{v}(t)?$
 c) $\vec{a}(t)?$
 d) $t = 30 \text{ s}$
 $\vec{v}(t)?$

$\vec{r} = v_0 \cdot t \cdot \vec{i} + \left(h - \frac{1}{2} g \cdot t^2 \right) \cdot \vec{j} =$
 $= 111 \cdot t \cdot \vec{i} + (2000 - 5 \cdot t^2) \cdot \vec{j}$

a) $y = 0 \Rightarrow 2000 - 5 \cdot t^2 = 0 \Rightarrow$
 $\Rightarrow t^2 = 400 \Rightarrow t = \boxed{20 \text{ s}}$

b) $x = v_0 \cdot t = 111 \cdot 20 = \boxed{2220 \text{ m}}$

c) $\vec{v} = 111 \cdot \vec{i} - 10 \cdot t \cdot \vec{j} = \frac{d\vec{r}}{dt}$

$\vec{v}(20) = 111 \cdot \vec{i} - 10 \cdot 20 \cdot \vec{j} = 111 \cdot \vec{i} - 200 \cdot \vec{j}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{111^2 + 200^2} = \boxed{229 \frac{\text{m}}{\text{s}}}$

d) $x = v_0 \cdot t = 111 \cdot 30 = \boxed{3330 \text{ m}}$

(29) $\alpha = 45^\circ$

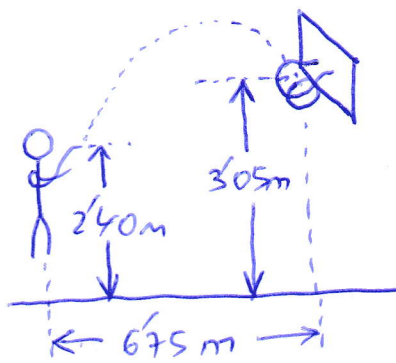
$x = 6.75 \text{ m}$

$h_0 = 2.40 \text{ m}$

$y = 3.05 \text{ m}$

$\vec{r}(t)?$

$\vec{v}(t)?$



$\vec{r} = v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} +$
 $+ \left(h + v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} g \cdot t^2 \right) \cdot \vec{j} =$
 $= v_0 \cdot \cos 45^\circ \cdot t \cdot \vec{i} +$
 $+ (2.40 + v_0 \cdot \sin 45^\circ \cdot t - 5t^2) \cdot \vec{j} =$
 $= v_0 \cdot 0.707 \cdot t \cdot \vec{i} +$
 $= (2.40 + v_0 \cdot 0.707 \cdot t - 5t^2) \cdot \vec{j}$

$x = 0.707 \cdot v_0 \cdot t = 6.75$

$y = 3.05 \Rightarrow 2.40 + 0.707 \cdot v_0 \cdot t - 5 \cdot t^2 = 3.05 \Rightarrow$

$\Rightarrow 2.40 + 6.75 - 5 \cdot t^2 = 3.05 \Rightarrow t = \boxed{1.11 \text{ s}}$

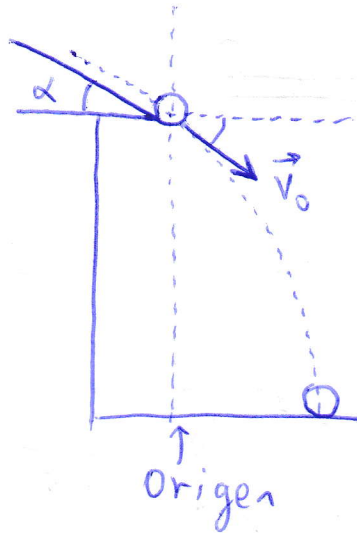
$v_0 = \frac{6.75}{0.707 \cdot 1.11} = \boxed{86 \frac{\text{m}}{\text{s}}}$

30) $h = 10 \text{ m}$

$\alpha = 30^\circ$

$v_0 = 2 \frac{\text{m}}{\text{s}}$

$r, v, A?$



$$\begin{aligned}\vec{r} &= v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \\ &+ \left(h - v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} = \\ &= 2 \cdot \cos 30^\circ \cdot t \cdot \vec{i} + \\ &+ \left(10 - 2 \cdot \sin 30^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} = \\ &= 1.73 \cdot t \cdot \vec{i} + \\ &+ (10 - t - 5t^2) \cdot \vec{j}\end{aligned}$$

* Tiempo de caída: $y = 0 \Rightarrow 10 - t - 5t^2 = 0 \Rightarrow$

$\Rightarrow -5t^2 - t + 10 = 0 \Rightarrow t = \frac{1 \pm \sqrt{1 + 4 \cdot 5 \cdot 10}}{2 \cdot (-5)} = \boxed{1.32 \text{ s}}$

* Velocidad al llegar al suelo:

$\vec{v} = \frac{d\vec{r}}{dt} = 1.73 \cdot \vec{i} + (-1 - 10 \cdot t) \cdot \vec{j}$

$\vec{v}(1.32) = 1.73 \cdot \vec{i} + (-1 - 10 \cdot 1.32) \cdot \vec{j} = 1.73 \cdot \vec{i} - 14.2 \cdot \vec{j}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{1.73^2 + 14.2^2} = \boxed{14.3 \frac{\text{m}}{\text{s}}}$

* Punto de impacto:

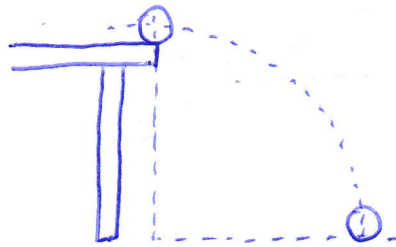
$x = v_0 \cdot \cos \alpha \cdot t = 1.73 \cdot t = 1.73 \cdot 1.32 = 2.28 \text{ m}$

$\boxed{P(2.28, 0) \text{ m}}$

(31) $h = 0.85 \text{ m}$

$v = 0.08 \frac{\text{m}}{\text{s}}$

$\vec{a}, t, v, P?$



$$\vec{r} = v_0 \cdot t \cdot \vec{i} + \left(h - \frac{1}{2} g \cdot t^2 \right) \cdot \vec{j}$$

$$\vec{r} = 0.08 \cdot t \cdot \vec{i} + (0.85 - 5 \cdot t^2) \cdot \vec{j}$$

$$y = 0 \Rightarrow 0.85 - 5 \cdot t^2 = 0 \Rightarrow t = \sqrt{\frac{0.85}{5}} = \boxed{0.412 \text{ s}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 0.08 \cdot \vec{i} - 10 \cdot t \cdot \vec{j}$$

$$\vec{v}(0.412) = 0.08 \cdot \vec{i} - 10 \cdot 0.412 \cdot \vec{j} = 0.08 \cdot \vec{i} - 4.12 \cdot \vec{j}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{0.08^2 + 4.12^2} = \boxed{4.12 \frac{\text{m}}{\text{s}}}$$

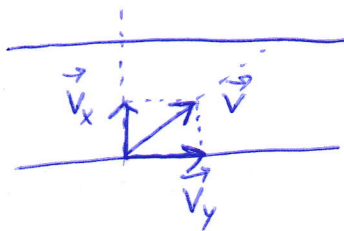
$$x = v_0 \cdot t = 0.08 \cdot 0.412 = 0.033 \text{ m} \Rightarrow \boxed{P(0.033, 0) \text{ m}}$$

(32) $a = 200 \text{ m}$

$v_y = 4 \frac{\text{m}}{\text{s}}$

$v_x = 3 \frac{\text{m}}{\text{s}}$

$\vec{a}, t, P, e?$



$$\vec{v} = \vec{v}_x + \vec{v}_y = 3 \cdot \vec{i} + 4 \cdot \vec{j}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \boxed{5 \frac{\text{m}}{\text{s}}}$$

$$v_y = \frac{a}{t} \Rightarrow t = \frac{a}{v_y} = \frac{200}{4} = \boxed{50 \text{ s}}$$

$$x = v_x \cdot t = 3 \cdot 50 = 150 \text{ m} \Rightarrow \boxed{P(150, 200) \text{ m}}$$

$$e = \sqrt{x^2 + y^2} = \sqrt{150^2 + 200^2} = \boxed{250 \text{ m}}$$

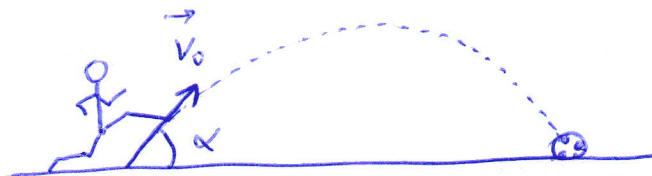
33) $v_0 = 25 \frac{m}{s}$

$\alpha = 30^\circ$

$\hat{r} h_{max}, t?$

$\hat{r} x?$

$\hat{r} v?$



$$\begin{aligned}\vec{r} &= v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} = \\ &= 25 \cdot \cos 30^\circ \cdot t \cdot \vec{i} + \left(25 \cdot \sin 30^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} = \\ &= 21.7 \cdot t \cdot \vec{i} + (12.5 \cdot t - 5 \cdot t^2) \cdot \vec{j}\end{aligned}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 21.7 \cdot \vec{i} + (12.5 - 10 \cdot t) \cdot \vec{j}$$

Altura máxima $\Rightarrow v_y = 0 \Rightarrow 12.5 - 10 \cdot t = 0 \Rightarrow t = \frac{12.5}{10} = \boxed{1.25 s}$

$h_{max} = 12.5 \cdot t - 5 \cdot t^2 = 12.5 \cdot 1.25 - 5 \cdot 1.25^2 = \boxed{7.81 m}$

Alcance: $t = 2 \cdot 1.25 = 2.5 s$

$x = 21.7 \cdot t = 21.7 \cdot 2.5 = \boxed{54.3 m}$

$\vec{v}(2.5) = 21.7 \cdot \vec{i} + (12.5 - 10 \cdot 2.5) \cdot \vec{j} = 21.7 \cdot \vec{i} - 12.5 \cdot \vec{j}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{21.7^2 + 12.5^2} = \boxed{25 \frac{m}{s}}$

(34) $\alpha = 45^\circ$ $\left| \begin{aligned} \vec{r} &= v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} = \\ h_{\max} &= 2400 \text{ m} \\ \hat{v}_0? \end{aligned} \right. \begin{aligned} &= v_0 \cdot \cos 45^\circ \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin 45^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} = \\ &= 0.707 \cdot v_0 \cdot t \cdot \vec{i} + (0.707 \cdot v_0 \cdot t - 5 \cdot t^2) \cdot \vec{j} \end{aligned}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 0.707 \cdot v_0 \cdot \vec{i} + (0.707 \cdot v_0 - 10 \cdot t) \cdot \vec{j}$$

Altura máxima $\Rightarrow v_y = 0 \Rightarrow 0.707 \cdot v_0 - 10 \cdot t = 0 \Rightarrow t = 0.707 \cdot v_0$

$y = 2400 \text{ m} \Rightarrow 0.707 \cdot v_0 \cdot t - 5 \cdot t^2 = 2400 \Rightarrow$

$\Rightarrow 0.707 \cdot v_0 \cdot 0.707 \cdot v_0 - 5 \cdot (0.707 \cdot v_0)^2 = 2400$

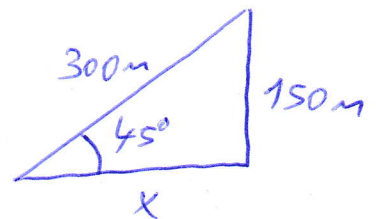
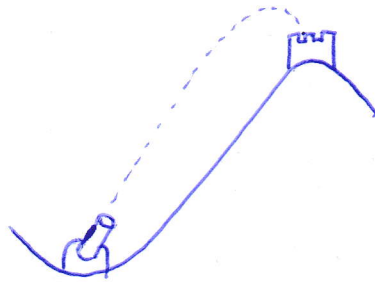
$0.5 \cdot v_0^2 - 0.25 \cdot v_0^2 = 2400 \Rightarrow v_0 = \boxed{310 \frac{\text{m}}{\text{s}}}$

(35) $y = 150 \text{ m}$

$d = 300 \text{ m}$

$\alpha = 45^\circ$

$\hat{t}, v_0?$



$\cos 45^\circ = \frac{x}{300} \Rightarrow x = 300 \cdot \cos 45^\circ = 212 \text{ m}$

$$\vec{r} = v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} =$$

$$= v_0 \cdot \cos 45^\circ \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin 45^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} =$$

$$= 0.707 \cdot v_0 \cdot t \cdot \vec{i} + (0.707 \cdot v_0 \cdot t - 5 \cdot t^2) \cdot \vec{j}$$

$x = 212 \text{ m} \Rightarrow 0.707 \cdot v_0 \cdot t = 212 \Rightarrow 212 - 5 \cdot t^2 = 150 \Rightarrow t = \boxed{3.52 \text{ s}}$

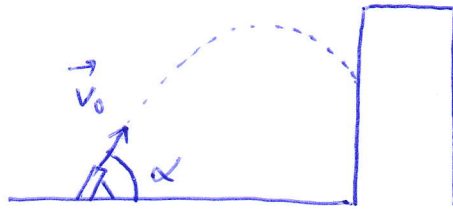
$v_0 = \frac{212}{0.707 \cdot 3.52} = \boxed{85.2 \frac{\text{m}}{\text{s}}}$

36 $\alpha = 60^\circ$

$y = 15 \text{ m}$

$x = 30 \text{ m}$

$\vec{r}, \vec{v}_0, \vec{v}?$



$$\begin{aligned}\vec{r} &= v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} = \\ &= v_0 \cdot \cos 60^\circ \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin 60^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} = \\ &= 0.5 \cdot v_0 \cdot t \cdot \vec{i} + (0.866 \cdot v_0 \cdot t - 5 \cdot t^2) \cdot \vec{j}\end{aligned}$$

$$x = 30 \text{ m} \Rightarrow 0.5 \cdot v_0 \cdot t = 30 \Rightarrow v_0 \cdot t = \frac{30}{0.5} = 60$$

$$y = 15 \Rightarrow 0.866 \cdot v_0 \cdot t - 5 \cdot t^2 = 15 \Rightarrow 0.866 \cdot 60 - 5 \cdot t^2 = 15 \Rightarrow$$

$$\Rightarrow t = \boxed{2.72 \text{ s}} ; v_0 = \frac{60}{t} = \frac{60}{2.72} = \boxed{22.1 \frac{\text{m}}{\text{s}}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 0.5 \cdot v_0 \cdot \vec{i} + (0.866 \cdot v_0 - 10 \cdot t) \cdot \vec{j} =$$

$$= 0.5 \cdot 22.1 \cdot \vec{i} + (0.866 \cdot 22.1 - 10 \cdot 2.72) \cdot \vec{j} =$$

$$= \boxed{11.1 \cdot \vec{i} - 8.06 \cdot \vec{j}}$$

37 a) MRU: $\vec{r} = \vec{r}_0 + \vec{v} \cdot t = 0 + 20 \cdot t \cdot \vec{j} = \boxed{+20 \cdot t \cdot \vec{j}}$

b) MRU: $\vec{r} = \vec{r}_0 + \vec{v} \cdot t$

$$\vec{v} = v_0 \cdot \cos \alpha \cdot \vec{i} + v_0 \cdot \sin \alpha \cdot \vec{j} = 20 \cdot \cos 45^\circ \cdot \vec{i} - 20 \cdot \sin 45^\circ \cdot \vec{j} = \boxed{14.1 \cdot \vec{i} - 14.1 \cdot \vec{j}}$$

c) $\vec{r} = \vec{r}_0 + \vec{v} \cdot t$; $\vec{r}_0 = 2\vec{i} + 6\vec{j}$; $\vec{v} = 20 \cdot t \cdot \vec{j}$

$$\boxed{\vec{r} = 2 \cdot \vec{i} + (6 + 20 \cdot t) \cdot \vec{j}}$$

d) MRUA: $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{g} \cdot t^2$

$$\vec{r}_0 = 100 \cdot \vec{j} ; \vec{v}_0 = -20 \cdot \vec{j} ; \vec{g} = -10 \cdot \vec{j}$$

$$\vec{r} = 100 \cdot \vec{j} - 20 \cdot \vec{j} \cdot t + \frac{1}{2} \cdot (-10 \cdot \vec{j}) \cdot t^2 = \boxed{(100 - 20 \cdot t - 5 \cdot t^2) \cdot \vec{j}}$$

38 a) MRUA: $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{g} \cdot t^2 =$

$$= \boxed{(50 - 55.6 \cdot t - 5 \cdot t^2) \cdot \vec{j}}$$

b) $\vec{r} = v_x \cdot t \cdot \vec{i} + v_y \cdot t \cdot \vec{j} = \boxed{3 \cdot t \cdot \vec{i} + 5 \cdot t \cdot \vec{j}}$

c) $h = 2000 \text{ m} ; v_0 = 139 \frac{\text{m}}{\text{s}}$

$$\vec{r} = v_0 \cdot t \cdot \vec{i} + \left(h - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} = \boxed{139 \cdot t \cdot \vec{i} + (2000 - 5 \cdot t^2) \cdot \vec{j}}$$

d) $v_0 = 55.6 \frac{\text{m}}{\text{s}}$

$$\vec{r} = v_0 \cdot \cos \alpha \cdot t \cdot \vec{i} + \left(v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \right) \cdot \vec{j} =$$

$$= 55.6 \cdot \cos 25^\circ \cdot t \cdot \vec{i} + \left(55.6 \cdot \sin 25^\circ \cdot t - \frac{1}{2} \cdot 10 \cdot t^2 \right) \cdot \vec{j} =$$

$$= \boxed{50.4 \cdot t \cdot \vec{i} + (23.5 \cdot t - 5 \cdot t^2) \cdot \vec{j}} \quad (38)$$

(39) $v_{01} = 50 \frac{m}{s}$

$\Delta t = 2 s$

$v_{02} = 80 \frac{m}{s}$

$\hat{r} t ? \hat{r} h ? \hat{r} v_1, v_2 ?$

MRUA: $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{g} \cdot t^2$

$\vec{r}_1 = (50 \cdot t - 5 \cdot t^2) \cdot \vec{j}$

$\vec{r}_2 = [80 \cdot (t-2) - 5 \cdot (t-2)^2] \cdot \vec{j} =$

$= [80 \cdot t - 160 - 5 \cdot (t^2 + 4 - 4t)] \cdot \vec{j} =$

$= (80 \cdot t - 160 - 5 \cdot t^2 - 20 + 20 \cdot t) = (-180 + 100 \cdot t - 5t^2) \cdot \vec{j}$

$\vec{r}_1 = \vec{r}_2 \Rightarrow 50 \cdot t - 5t^2 = -180 + 100 \cdot t - 5t^2 \Rightarrow$

$\Rightarrow 180 = 100 \cdot t - 50 \cdot t \Rightarrow 180 = 50 \cdot t \Rightarrow t = \boxed{36 s}$

$\vec{r} = (50 \cdot t - 5 \cdot t^2) \cdot \vec{j} = (50 \cdot 36 - 5 \cdot 36^2) \cdot \vec{j} = 115 \cdot \vec{j} \Rightarrow h = \boxed{115 m}$

$\vec{v}_1 = \frac{d\vec{r}_1}{dt} = (50 - 10 \cdot t) \cdot \vec{j} = (50 - 10 \cdot 36) \cdot \vec{j} = 14 \cdot \vec{j} \Rightarrow v_1 = \boxed{14 \frac{m}{s}}$

$\vec{v}_2 = \frac{d\vec{r}_2}{dt} = (100 - 10 \cdot t) \cdot \vec{j} = (100 - 10 \cdot 36) \cdot \vec{j} = 64 \cdot \vec{j} \Rightarrow v_2 = \boxed{64 \frac{m}{s}}$

(40) $h = 10 m$

$v_0 = 15 \frac{m}{s}$

$\hat{r} t, h ?$

$\vec{r}_1 = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{g} \cdot t^2 =$

$= (10 + 0 - \frac{1}{2} \cdot 10 \cdot t^2) \cdot \vec{j} = (10 - 5 \cdot t^2) \cdot \vec{j}$

$\vec{r}_2 = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{g} \cdot t^2 = (0 + 15 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2) \cdot \vec{j} = (15 \cdot t - 5 \cdot t^2) \cdot \vec{j}$

$\vec{r}_1 = \vec{r}_2 \Rightarrow 10 - 5t^2 = 15 \cdot t - 5t^2 \Rightarrow t = \frac{10}{15} = \boxed{0.667 s}$

$\vec{r}_1 = (10 - 5 \cdot t^2) \cdot \vec{j} = (10 - 5 \cdot 0.667^2) \cdot \vec{j} = 7.78 \cdot \vec{j} \Rightarrow h = \boxed{7.78 m}$

$$\textcircled{41} \quad v_0 = 556 \frac{\text{m}}{\text{s}} \quad \left| \quad \vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{g} \cdot t^2 = \right.$$

$$\hat{r}_{\text{máx}}, t_T, v? \quad \left| \quad = 0 + 556 \cdot \vec{j} \cdot t + \frac{1}{2} \cdot (-10 \cdot \vec{j}) \cdot t^2 = \right.$$

$$= \boxed{(556 \cdot t - 5 \cdot t^2) \cdot \vec{j}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (556 - 10 \cdot t) \cdot \vec{j}$$

$$\text{Altura máxima} \Rightarrow v = 0 \Rightarrow 556 - 10 \cdot t = 0 \Rightarrow t = 556 \text{ s}$$

$$y = 556 \cdot t - 5 \cdot t^2 = 556 \cdot 556 - 5 \cdot 556^2 = \boxed{155 \text{ m}}$$

$$t_T = 2 \cdot 556 = \boxed{1111 \text{ s}}$$

$$\vec{v}(1111) = (556 - 10 \cdot 1111) \cdot \vec{j} = -554 \cdot \vec{j} \Rightarrow v = \boxed{554 \frac{\text{m}}{\text{s}}}$$

$$\textcircled{42} \quad v = 3 \frac{\text{m}}{\text{s}} \quad \left| \quad \boxed{\vec{r}_1 = 3 \cdot t \cdot \vec{j}} \right.$$

$$\hat{t}?$$

$$h_{\text{ascensor}} = 25 \text{ m}$$

$$\frac{d\vec{r}_1, \vec{r}_2?}{\left| \quad \vec{r}_2 = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{g} \cdot t^2 = \right.}$$

$$= \boxed{(25 + 3 \cdot t - 5 \cdot t^2) \cdot \vec{j}}$$

Cuando choca contra el suelo:

$$\vec{r}_1 = \vec{r}_2 \Rightarrow 3 \cdot t = 25 + 3 \cdot t - 5 \cdot t^2 \Rightarrow 5 \cdot t^2 = 25 \Rightarrow$$

$$\Rightarrow t = \boxed{0.707 \text{ s}}$$

(43) $v = 5 \frac{m}{s}$

$h = 120 \text{ m}$

$\vec{r}?$, $\vec{v}?$

$\vec{r}?$

MARUA: $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \cdot \vec{g} \cdot t^2 =$

$= (120 + 5 \cdot t - 5 \cdot t^2) \cdot \vec{j}$

En el suelo: $\vec{r} = 0 \Rightarrow 120 + 5t - 5t^2 = 0 \Rightarrow$

$\Rightarrow -5t^2 + 5t + 120 = 0$

$t = \frac{-5 \pm \sqrt{5^2 + 4 \cdot 5 \cdot 120}}{2 \cdot (-5)} = \boxed{5.42 \text{ s}}$

$\vec{v} = \frac{d\vec{r}}{dt} = (5 - 10 \cdot t) \cdot \vec{j}$

$\vec{v}(5.42) = (5 - 10 \cdot 5.42) \cdot \vec{j} = -49.2 \cdot \vec{j} \Rightarrow \boxed{v = 49.2 \frac{m}{s}}$

(44) $\alpha = 90^\circ$

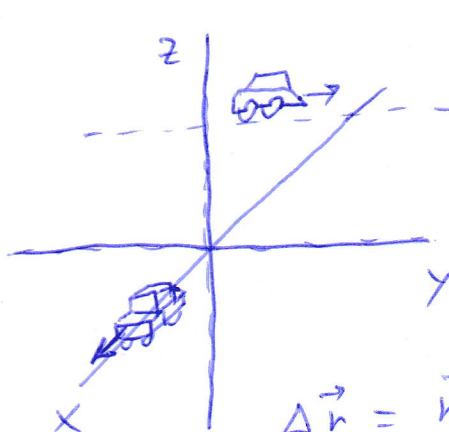
$v = 4 \frac{m}{s}$

$v = 3 \frac{m}{s}$

$h = 11 \text{ m}$

$\vec{d}?$, $t = 10 \text{ s}$

$\vec{v}?$



$\vec{r}_1 = 3 \cdot t \cdot \vec{i}$

$\vec{r}_2 = 4 \cdot t \cdot \vec{j} + 11 \cdot \vec{k}$

$\vec{v}_1 = 3 \cdot \vec{i}$

$\vec{v}_2 = 4 \cdot \vec{j}$

$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 =$

$= 4 \cdot t \cdot \vec{j} + 11 \cdot \vec{k} - 3 \cdot t \cdot \vec{i} = -3 \cdot t \cdot \vec{i} + 4 \cdot t \cdot \vec{j} + 11 \cdot \vec{k}$

$\Delta \vec{r}(t=10s) = -30 \cdot \vec{i} + 40 \cdot \vec{j} + 11 \cdot \vec{k}$

$d = |\Delta \vec{r}| = \sqrt{30^2 + 40^2 + 11^2} = \boxed{51.2 \text{ m}}$

$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 = 4 \cdot \vec{j} - 3 \cdot \vec{i} = -3 \cdot \vec{i} + 4 \cdot \vec{j}$

$v = |\Delta \vec{v}| = \sqrt{3^2 + 4^2} = \boxed{5 \frac{m}{s}}$

$$\textcircled{45} \quad v = 3 \frac{\text{m}}{\text{s}} \quad \left| \quad \text{MRU: } \vec{r} = \vec{r}_1 + \vec{r}_2 \right.$$

$$v = 0.9 \frac{\text{m}}{\text{s}} \quad \left| \quad \begin{aligned} \vec{r}_1 &= 3 \cdot t \cdot \vec{j} ; \vec{r}_2 = 0.9 \cdot t \cdot \vec{i} \\ \vec{r} &= 0.9 \cdot t \cdot \vec{i} + 3 \cdot t \cdot \vec{j} \end{aligned} \right.$$

$$\vec{r}(3) = 0.9 \cdot 3 \cdot \vec{i} + 3 \cdot 3 \cdot \vec{j} = 2.7 \cdot \vec{i} + 9 \cdot \vec{j}$$

$$d = \sqrt{x^2 + y^2} = \sqrt{2.7^2 + 9^2} = \boxed{9.4 \text{ m}}$$

$$\textcircled{46} \quad \vec{r}_1 = 41.7 \cdot \cos 40^\circ \cdot t \cdot \vec{i} + (41.7 \cdot \sin 40^\circ \cdot t - 5 \cdot t^2) \cdot \vec{j} =$$

$$= \boxed{31.9 \cdot t \cdot \vec{i} + (26.8 \cdot t - 5 \cdot t^2) \cdot \vec{j}}$$

$$\vec{r}_2 = x \cdot \vec{i} + \left(h - \frac{1}{2} \cdot g \cdot t^2\right) \cdot \vec{j} = x \cdot \vec{i} + (50 - 5 \cdot t^2) \cdot \vec{j}$$

$$\vec{r}_1 = \vec{r}_2 \Rightarrow \left\{ \begin{aligned} x &= 31.9 \cdot t \\ 26.8 \cdot t - 5 \cdot t^2 &= 50 - 5 \cdot t^2 \end{aligned} \right\} \quad t = \frac{50}{26.8} = \boxed{1.87 \text{ s}}$$

$$x = 31.9 \cdot 1.87 = \boxed{59.7 \text{ m}}$$

$$\vec{r}_2 = \boxed{59.7 \cdot \vec{i} + (50 - 5 \cdot t^2) \cdot \vec{j}}$$

$$y = 50 - 5 \cdot t^2 = 50 - 5 \cdot 1.87^2 = 32.5 \text{ m}$$

$$\text{Lugar de impacto: } \boxed{P(59.7, 32.5) \text{ m}}$$

PROBLEMAS RESUELTOS DE DINÁMICA

① ¿ F ?

$$d = 3'8 \cdot 10^8 \text{ m}$$

$$G = 6'67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$M_T = 5'97 \cdot 10^{24} \text{ kg}$$

$$M_L = 7'35 \cdot 10^{22} \text{ kg}$$

$$F = G \cdot \frac{M_T \cdot M_L}{r^2} =$$

$$= 6'67 \cdot 10^{-11} \cdot \frac{5'97 \cdot 10^{24} \cdot 7'35 \cdot 10^{22}}{(3'8 \cdot 10^8)^2} =$$

$$= \boxed{2'03 \cdot 10^{20} \text{ N}}$$

② ¿ $\frac{F_E}{F_G}$?

$$m_p = 1'67 \cdot 10^{-27} \text{ kg}$$

$$m_e = 9'1 \cdot 10^{-31} \text{ kg}$$

$$e = 1'6 \cdot 10^{-19} \text{ C}$$

$$G = 6'67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$\frac{F_E}{F_G} = \frac{k \cdot \frac{Q_1 \cdot Q_2}{r^2}}{G \cdot \frac{m_p \cdot m_e}{r^2}} =$$

$$= \frac{k \cdot e^2}{G \cdot m_p \cdot m_e} =$$

$$= \frac{9 \cdot 10^9 \cdot (1'6 \cdot 10^{-19})^2}{6'67 \cdot 10^{-11} \cdot 1'67 \cdot 10^{-27} \cdot 9'1 \cdot 10^{-31}} =$$

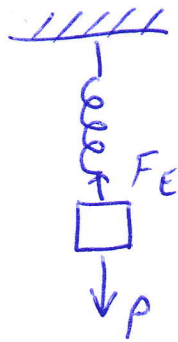
$$= \boxed{2'27 \cdot 10^{39}}$$

③ $l_0 = 0.2 \text{ m}$

$k = 60 \frac{\text{N}}{\text{m}}$

$m = 0.3 \text{ kg}$

¿l?



$F_E = P$

$k \cdot x = m \cdot g$

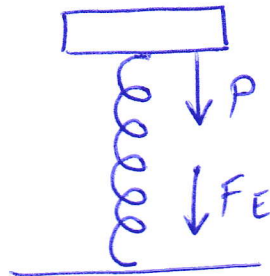
$x = \frac{m \cdot g}{k} = \frac{0.3 \cdot 10}{60} = \frac{3}{60} = 0.05 \text{ m}$

$l = l_0 + x = 0.2 + 0.05 = \boxed{0.25 \text{ m}}$

④ $k = 167 \frac{\text{N}}{\text{m}}$

$m = 0.5 \text{ kg}$

¿x?



$P = F_E$

$m \cdot g = k \cdot x$

$x = \frac{m \cdot g}{k} = \frac{0.5 \cdot 10}{167} = 0.03 \text{ m} = \boxed{3 \text{ cm}}$

⑤ ¿E?

$V_S = 777 \text{ L} = 0.777 \text{ m}^3$

$d_F = 1030 \frac{\text{kg}}{\text{m}^3}$

¿m?

$E = d_F \cdot V_S \cdot g = 1030 \cdot 0.777 \cdot 10 = \boxed{8000 \text{ N}}$

$E = P = m \cdot g \Rightarrow$

$\Rightarrow m = \frac{E}{g} = \frac{8000}{10} = \boxed{800 \text{ N}}$

⑥ ¿P_{ap}?

$$m = 85 \text{ kg}$$

$$V_s = 90 \text{ l}$$

¿Porcentaje?

$$P_{ap} = P - E = m \cdot g - d_F \cdot V_s \cdot g =$$

$$= 85 \cdot 10 - 1000 \cdot 90 \cdot 10^{-3} \cdot 10 =$$

$$= 850 - 900 = \boxed{-50 \text{ N}}$$

* Condición de flotabilidad: $E = P \Rightarrow$

$$\Rightarrow d_F \cdot V_s \cdot g = m \cdot g \Rightarrow V_s = \frac{m}{d_F} = \frac{85}{1000} = 0.085 \text{ m}^3 =$$

$$= 85 \text{ l}$$

$$\text{Porcentaje sumergido} = \frac{85 \cdot 100}{90} = \boxed{94\frac{4}{9}\%}$$

⑦ ¿F?

$$k = 80 \frac{\text{N}}{\text{m}}$$

$$l = 0.25 \text{ m}$$

$$l_0 = 0.20 \text{ m}$$

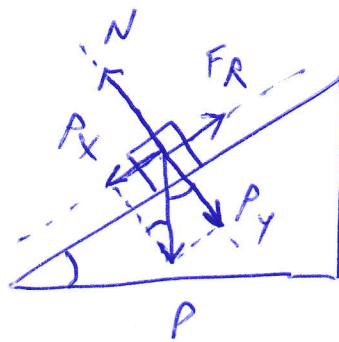
$$F = k \cdot |l - l_0| = 80 \cdot |0.25 - 0.20| =$$

$$= 80 \cdot 0.05 = \boxed{4 \text{ N}}$$

8 $v_0 = 0$

$\alpha = 52^\circ$

$\hat{c} \mu_E?$



$R = 0 \Rightarrow P_x = F_R$

$$m \cdot g \cdot \sin \alpha = \mu_E \cdot m \cdot g \cdot \cos \alpha \Rightarrow \mu_E = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha =$$

$= \tan 52^\circ = \boxed{1.28}$

9 $\mu_E = 0.45$

$\mu_D = 0.3$

$m = 60 \text{ kg}$

$\hat{c} F_R?$

$F_{R1} = \mu_E \cdot m \cdot g = 0.45 \cdot 60 \cdot 10 = \boxed{270 \text{ N}}$

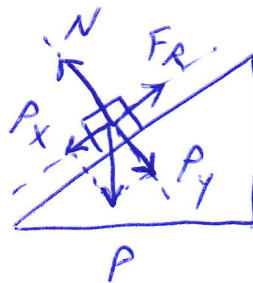
$F_{R2} = \mu_D \cdot m \cdot g = 0.3 \cdot 60 \cdot 10 = \boxed{180 \text{ N}}$

10 $m = 80 \text{ kg}$

$\alpha = 30^\circ$

$\mu = 0.25$

$\hat{c} \text{ Fuerzas?}$



$P = m \cdot g = 80 \cdot 10 = \boxed{800 \text{ N}}$

$P_x = m \cdot g \cdot \sin \alpha = 80 \cdot 10 \cdot \sin 30^\circ = \boxed{400 \text{ N}}$

$P_y = N = m \cdot g \cdot \cos \alpha = 80 \cdot 10 \cdot \cos 30^\circ = \boxed{693 \text{ N}}$

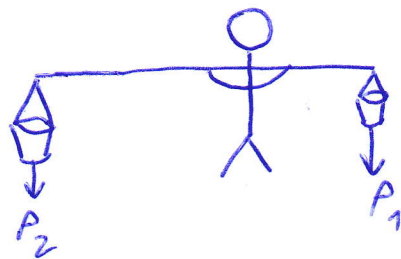
$F_R = \mu \cdot N = 0.25 \cdot 693 = \boxed{173 \text{ N}}$

11 $l = 2 \text{ m}$

$m_1 = 8 \text{ kg}$

$m_2 = 12 \text{ kg}$

$\hat{r} x_1, x_2?$



$F = P_1 + P_2 =$

$= m_1 \cdot g + m_2 \cdot g =$

$= 8 \cdot 10 + 12 \cdot 10 = 80 + 120 = \boxed{200 \text{ N}}$

$F_1 \cdot x_1 = F_2 \cdot x_2 \Rightarrow P_1 \cdot x_1 = P_2 \cdot (l - x_1) \Rightarrow$

$\Rightarrow 80 \cdot x_1 = 120 \cdot (2 - x_1) \Rightarrow 80 \cdot x_1 = 240 - 120 \cdot x_1 \Rightarrow$

$\Rightarrow 200 \cdot x_1 = 240 \Rightarrow x_1 = \frac{240}{200} = \boxed{1.2 \text{ m}}$

$x_2 = 2 - x_1 = 2 - 1.2 = \boxed{0.8 \text{ m}}$

12 $l = 0.6 \text{ m}$

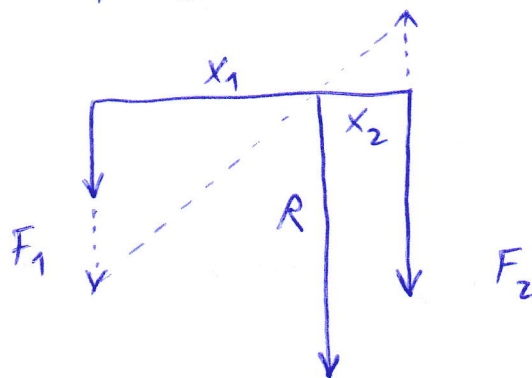
$\hat{r} R?$

$\hat{r} x_1, x_2?$

$F_1 = 250 \text{ N}$

$F_2 = 320 \text{ N}$

$R = F_1 + F_2 = 250 + 320 = \boxed{570 \text{ N}}$



$F_1 \cdot x_1 = F_2 \cdot x_2 \Rightarrow F_1 \cdot x_1 = F_2 \cdot (l - x_1) \Rightarrow$

$\Rightarrow 250 \cdot x_1 = 320 \cdot (0.6 - x_1) \Rightarrow 250 \cdot x_1 = 192 - 320 \cdot x_1 \Rightarrow$

$\Rightarrow 570 \cdot x_1 = 192 \Rightarrow x_1 = \frac{192}{570} = \boxed{0.337 \text{ m}}$

$x_2 = \boxed{0.263 \text{ m}}$

(13) $F_1 = 300 \text{ N}$

$F_2 = 400 \text{ N}$

$\alpha = 30^\circ$

$\vec{R}?$

$$R = \sqrt{F_1^2 + F_2^2 - 2 \cdot F_1 \cdot F_2 \cdot \cos \alpha} =$$

$$= \sqrt{300^2 + 400^2 - 2 \cdot 300 \cdot 400 \cdot \cos 30^\circ} =$$

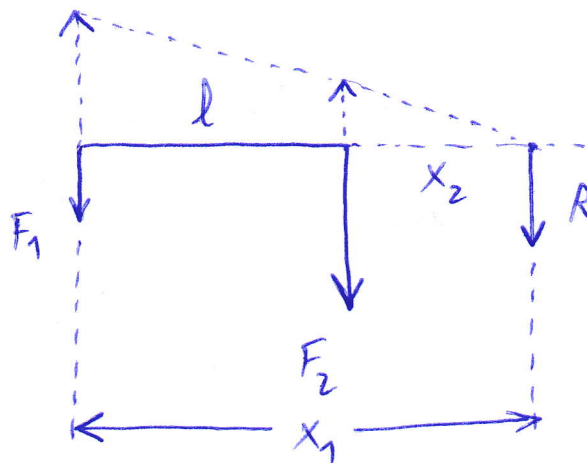
$$= \boxed{205 \text{ N}}$$

(14) $F_1 = 120 \text{ N}$

$F_2 = 200 \text{ N}$

$l = 1/5 \text{ m}$

$\vec{R}, x_1, x_2?$



$$R = F_2 - F_1 = 200 - 120 = \boxed{80 \text{ N}}$$

$$F_1 \cdot x_1 = F_2 \cdot x_2 \Rightarrow F_1 \cdot (l + x_2) = F_2 \cdot x_2 \Rightarrow$$

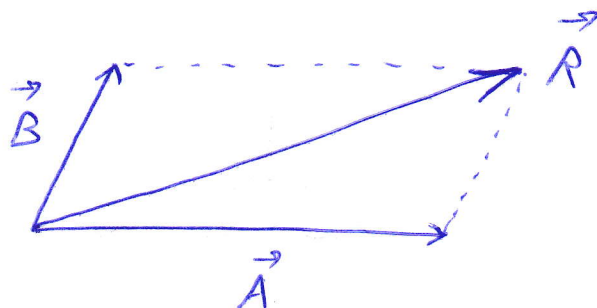
$$\Rightarrow 120 \cdot (1/5 + x_2) = 200 \cdot x_2 \Rightarrow$$

$$\Rightarrow 180 + 120 \cdot x_2 = 200 \cdot x_2 \Rightarrow 180 = 80 \cdot x_2 \Rightarrow$$

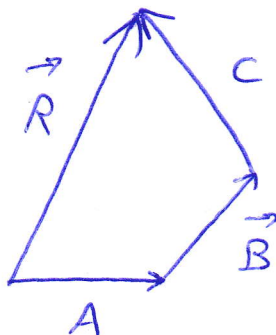
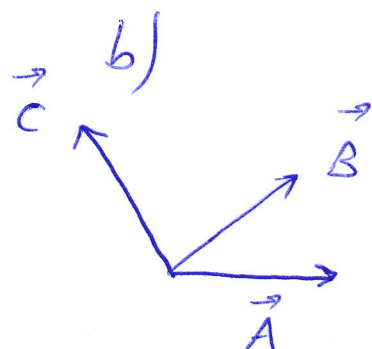
$$\Rightarrow x_2 = \frac{180}{80} = \boxed{2.25 \text{ m}}$$

$$x_1 = l + x_2 = 1/5 + 2.25 = \boxed{3.75 \text{ m}}$$

15 a)



Método del paralelogramo

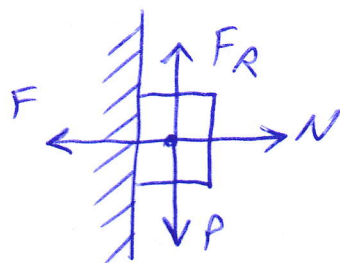


Método del polígono

16 $m = 50 \text{ kg}$

$\hat{a} F?$

$\mu = 0.33$



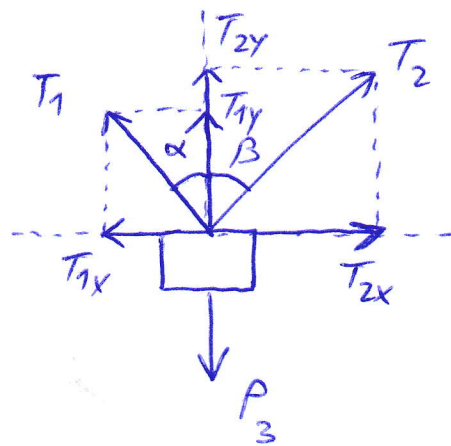
* Condición de estática:

$$\sum \vec{F} = 0 \Rightarrow \left\{ \begin{array}{l} \sum F_x = 0 \Rightarrow F = N \\ \sum F_y = 0 \Rightarrow F_R = P \end{array} \right\}$$

$$\mu \cdot N = m \cdot g \Rightarrow N = \frac{m \cdot g}{\mu} = \frac{50 \cdot 10}{0.33} = 1515 \text{ N}$$

$$F = N = \boxed{1515 \text{ N}}$$

17) $m_1 = 20 \text{ kg}$
 $m_2 = 24'5 \text{ kg}$
 $\alpha = 30^\circ$
 $\hat{=} m_3, \beta?$



* Condición de estática:

$$\sum \vec{F} = 0 \Rightarrow \left\{ \begin{array}{l} \sum \vec{F}_x = 0 \Rightarrow T_{1x} = T_{2x} \\ \sum \vec{F}_y = 0 \Rightarrow T_{1y} + T_{2y} = P_3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} T_1 \cdot \sin \alpha = T_2 \cdot \sin \beta \\ T_1 \cdot \cos \alpha + T_2 \cdot \cos \beta = m_3 \cdot g \end{array} \right\}$$

Por otro lado: $T_1 = m_1 \cdot g = 20 \cdot 10 = 200 \text{ N}$

$T_2 = m_2 \cdot g = 24'5 \cdot 10 = 245 \text{ N}$

$$\sin \beta = \frac{T_1 \cdot \sin \alpha}{T_2} = \frac{200 \cdot \sin 30^\circ}{245} = 0'408 \Rightarrow \boxed{\beta = 24'1^\circ}$$

$$m_3 = \frac{T_1 \cdot \cos \alpha + T_2 \cdot \cos \beta}{g} =$$

$$= \frac{200 \cdot \cos 30^\circ + 245 \cdot \cos 24'1^\circ}{10} =$$

$$= \boxed{39'7 \text{ kg}}$$

18) $m = 0.024 \text{ kg}$

$Q_1 = 6 \cdot 10^{-3} \text{ C}$

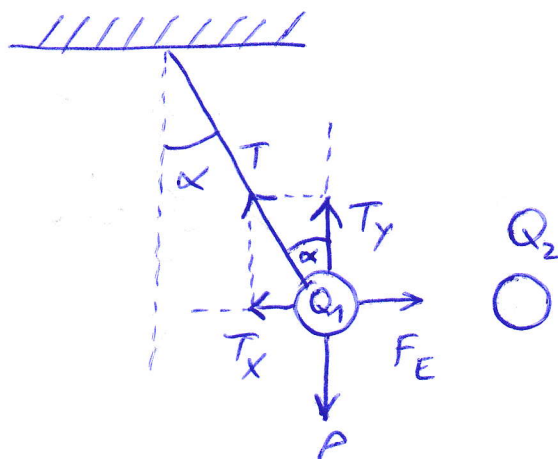
$\alpha = 30^\circ$

$Q_2 = ?$

$T = ?$

$r = 0.5 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



* Condición de estática:

$$\sum \vec{F} = 0 \Rightarrow \begin{cases} \sum F_x = 0 \Rightarrow T_x = F_E \\ \sum F_y = 0 \Rightarrow T_y = P \end{cases} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} T \cdot \sin \alpha &= k \cdot \frac{Q_1 \cdot Q_2}{r^2} \\ T \cdot \cos \alpha &= m \cdot g \end{aligned} \right\} \quad T = \frac{m \cdot g}{\cos \alpha} = \frac{0.024 \cdot 10}{\cos 30^\circ} = \boxed{0.277 \text{ N}}$$

$$|Q_2| = \frac{T \cdot \sin \alpha \cdot r^2}{k \cdot Q_1} = \frac{0.277 \cdot \sin 30^\circ \cdot 0.5^2}{9 \cdot 10^9 \cdot 6 \cdot 10^{-3}} = 6.41 \cdot 10^{-10} \text{ C}$$

$$Q_2 = \boxed{-6.41 \cdot 10^{-10} \text{ C}}$$

Para que ambas cargas se atraigan, deben ser de signos opuestos.

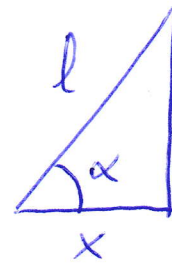
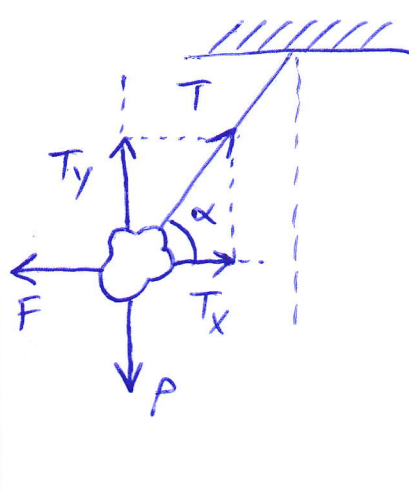
19) $m = 62 \text{ kg}$

¿F?

$x = 20 \text{ m}$

$l = 25 \text{ m}$

¿T?



* Condición de estática:

$$\sum \vec{F} = 0 \Rightarrow \left\{ \begin{array}{l} \sum \vec{F}_x = 0 \Rightarrow F = T_x \\ \sum \vec{F}_y = 0 \Rightarrow T_y = P \end{array} \right\} \begin{array}{l} F = T \cdot \cos \alpha \\ T \cdot \sin \alpha = m \cdot g \end{array}$$

$$\cos \alpha = \frac{x}{l} = \frac{20}{25} = 0.8 \Rightarrow \alpha = 36.9^\circ$$

$$T = \frac{m \cdot g}{\sin \alpha} = \frac{62 \cdot 10}{\sin 36.9^\circ} = \boxed{1033 \text{ N}}$$

$$F = T \cdot \cos \alpha = 1033 \cdot \cos 36.9^\circ = \boxed{826 \text{ N}}$$

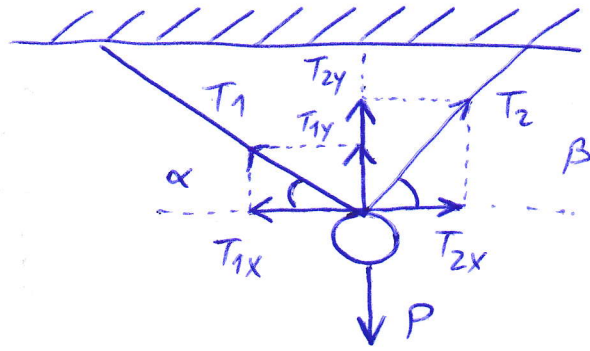
20) $m = 60 \text{ kg}$

$l_1 = 3 \text{ m}$

$l_2 = 2 \text{ m}$

$\alpha, T_1, T_2?$

$\alpha = 30^\circ$



$\alpha + \beta + 90^\circ = 180^\circ \Rightarrow \beta = 180 - 90 - 30 = 60^\circ$

* Condición de estática:

$$\sum \vec{F} = 0 \Rightarrow \left\{ \begin{array}{l} \sum \vec{F}_x = 0 \Rightarrow T_{1x} = T_{2x} \\ \sum \vec{F}_y = 0 \Rightarrow T_{1y} + T_{2y} = P \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} T_1 \cdot \cos \alpha = T_2 \cdot \cos \beta \\ T_1 \cdot \sin \alpha + T_2 \cdot \sin \beta = m \cdot g \end{array} \right\} \Rightarrow \left. \begin{array}{l} T_1 \cdot \cos 30^\circ = T_2 \cdot \cos 60^\circ \\ T_1 \cdot \sin 30^\circ + T_2 \cdot \sin 60^\circ = 60 \cdot 10 \end{array} \right\} \Rightarrow$$

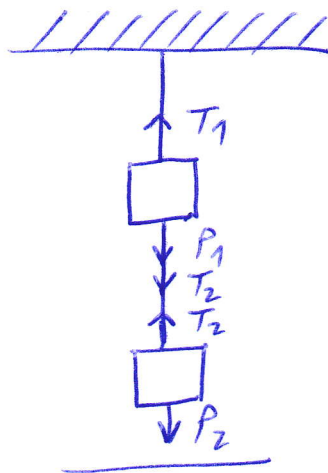
$$\Rightarrow \left. \begin{array}{l} 0'866 \cdot T_1 = 0'5 \cdot T_2 \\ 0'5 \cdot T_1 + 0'866 \cdot T_2 = 600 \end{array} \right\} T_1 = \frac{0'5 \cdot T_2}{0'866} = 0'577 \cdot T_2$$

$0'5 \cdot 0'577 \cdot T_2 + 0'866 \cdot T_2 = 600 \Rightarrow$

$\Rightarrow T_2 = \frac{600}{0'5 \cdot 0'577 + 0'866} = \boxed{520 \text{ N}}$

$T_1 = 0'577 \cdot T_2 = 0'577 \cdot 520 = \boxed{300 \text{ N}}$

21 $m_1 = 8 \text{ kg}$
 $m_2 = 6 \text{ kg}$
 ¿ T_1, T_2 ?



* Condición de estática: $\sum \vec{F} = 0 \Rightarrow \begin{cases} T_1 = P_1 + T_2 \\ T_2 = P_2 \end{cases}$

$T_2 = P_2 = m_2 \cdot g = 6 \cdot 10 = \boxed{60 \text{ N}}$

$T_1 = P_1 + T_2 = m_1 \cdot g + T_2 = 8 \cdot 10 + 60 = \boxed{140 \text{ N}}$

22 $F = 20 \text{ N}$ $M_1 = F \cdot h_1 = 20 \cdot 0.1 = \boxed{2 \text{ N} \cdot \text{m}}$

$h_1 = 0.1 \text{ m}$

$h_2 = 1 \text{ m}$

¿ M_1, M_2 ?

$M_2 = F \cdot h_2 = 20 \cdot 1 = \boxed{20 \text{ N} \cdot \text{m}}$

(23) $F_1 = 0.05 \cdot P$

$r_1 = 0.8 \text{ m}$

$\hat{c} F?$

$m = 12 \text{ kg}$

$r_2 = 0.5 \text{ m}$

$r_3 = 0.1 \text{ m}$

* Momento necesario para abrirla:

$M = r_1 \cdot F_1 = r_1 \cdot 0.05 \cdot P =$

$= 0.8 \cdot 0.05 \cdot 12 \cdot 10 = 4.8 \text{ N} \cdot \text{m}$

$M_2 = r_2 \cdot F_2 = M \Rightarrow$

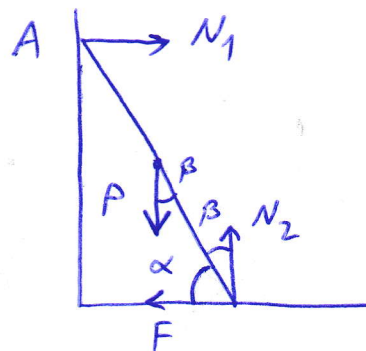
$\Rightarrow F_2 = \frac{M}{r_2} = \frac{4.8}{0.5} = \boxed{9.6 \text{ N}}$

$M_3 = r_3 \cdot F_3 = M \Rightarrow F_3 = \frac{M}{r_3} = \frac{4.8}{0.1} = \boxed{48 \text{ N}}$

(24) $l = 3 \text{ m}$

$\alpha = 60^\circ$

$\hat{c} F?$



$\beta = 90^\circ - \alpha = 90 - 60 = 30^\circ$

$\sin \alpha = \cos \beta$

$\cos \alpha = \sin \beta$

$M = r \cdot F \cdot \sin \varphi$; φ es el ángulo entre r y F .

Criterio de signos: los momentos hacia arriba y hacia la derecha son positivos; hacia la izquierda y hacia abajo, negativos.

$\sum \vec{F}_y = 0 \Rightarrow P = N_2 = m \cdot g$

Tomando momentos respecto al punto A:

$-\frac{l}{2} \cdot P \cdot \sin \beta - F \cdot l \cdot \sin \alpha + N_2 \cdot l \cdot \sin \beta = 0$

$-\frac{3}{2} \cdot 40 \cdot 10 \cdot \sin 30^\circ - F \cdot 3 \cdot \sin 60^\circ + 40 \cdot 10 \cdot 3 \cdot \sin 30^\circ = 0$

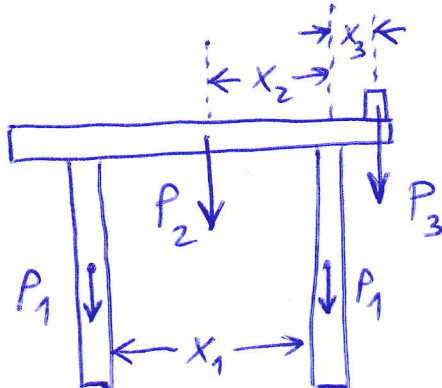
$-300 - 2.6 \cdot F + 600 = 0 \Rightarrow F = \boxed{115 \text{ N}}$

(55)

25) ¿m?

$$m_2 = 12 \text{ kg}$$

$$m_1 = 5 \text{ kg}$$



$$P_1 = 5 \cdot 10 = 50 \text{ N}; \quad P_2 = m_2 \cdot g = 12 \cdot 10 = 120 \text{ N}$$

Tomando momentos respecto a la pata derecha:

$$P_1 \cdot x_1 + P_2 \cdot x_2 = P_3 \cdot x_3; \quad x_3 = 0.15 \text{ m};$$

$$x_1 = 1.8 - 0.15 \cdot 2 = 1.5 \text{ m}; \quad x_2 = \frac{x_1}{2} = 0.75 \text{ m}$$

$$50 \cdot 1.5 + 120 \cdot 0.75 = m_3 \cdot 10 \cdot 0.15 \Rightarrow m_3 = \boxed{110 \text{ kg}}$$

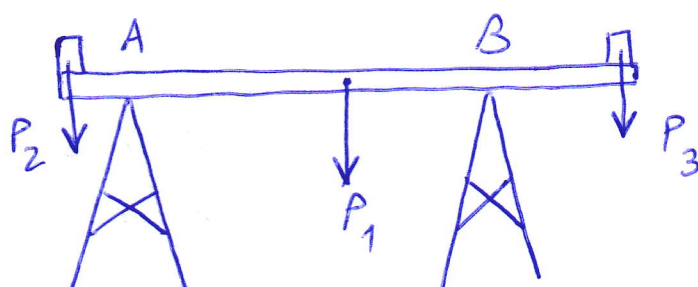
26) $m_1 = 6 \text{ kg}$

$$l = 2.4 \text{ m}$$

$$x_1 = 0.8 \text{ m}$$

$$x_2 = 0.2 \text{ m}$$

¿ m_2, m_3 ?



* Tomando momentos respecto a A:

$$P_2 \cdot x_2 = P_1 \cdot x_1; \quad x_1 = 0.8 \text{ m}$$

$$x_2 = \frac{2.4}{2} - 0.8 = 1.2 - 0.8 = 0.4 \text{ m}$$

$$m_2 \cdot 10 \cdot 0.4 = 6 \cdot 10 \cdot 0.8 \Rightarrow m_2 = \frac{6 \cdot 0.8}{0.4} = \boxed{12 \text{ kg}}$$

* Tomando momentos respecto a B:

$$P_1 \cdot x_1 = P_3 \cdot x_3; \quad x_1 = 0.2 \text{ m}; \quad x_3 = \frac{2.4}{2} - 0.2 = 1.2 - 0.2 = 1 \text{ m}$$

$$6 \cdot 10 \cdot 0.2 = m_3 \cdot 10 \cdot 1 \Rightarrow m_3 = \boxed{1.2 \text{ kg}}$$

27 $m_1 = 20 \text{ kg}$

$r_1 = 2 \text{ m}$

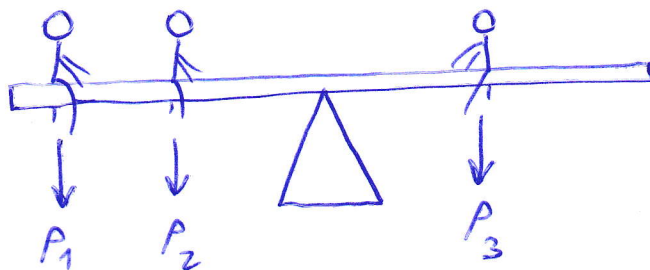
$m_2 = 25 \text{ kg}$

$r_2 = 0.8 \text{ m}$

$m_3 = 41 \text{ kg}$

$r_3 = 1.5 \text{ m}$

¿M?



* Condición de equilibrio:

$$P_1 \cdot x_1 + P_2 \cdot x_2 = P_3 \cdot x_3$$

$$M_{\text{izquierda}} = P_1 \cdot x_1 + P_2 \cdot x_2 =$$

$$= 20 \cdot 10 \cdot 2 + 25 \cdot 10 \cdot 0.8 = 600 \text{ N}\cdot\text{m}$$

$$M_{\text{derecha}} = P_3 \cdot x_3 = 41 \cdot 10 \cdot 1.5 = 615 \text{ N}\cdot\text{m}$$

Al ser: $M_{\text{derecha}} > M_{\text{izquierda}}$, el sistema rota en sentido horario.

$$M_{\text{total}} = M_{\text{derecha}} - M_{\text{izquierda}} = 615 - 600 = \boxed{15 \text{ N}\cdot\text{m}}$$

28 $a = 0.6 \text{ m}$

$b = 0.8 \text{ m}$

$\mu = 0.8$

¿v? $\rightarrow \frac{\text{km}}{\text{h}}$

$R = 50 \text{ m}$

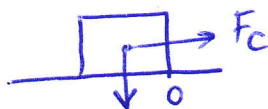
¿v? $\rightarrow \frac{\text{km}}{\text{h}}$

* Velocidad a la que se sale:

$$F_c = F_R \Rightarrow \frac{m \cdot v^2}{R} = \mu \cdot m \cdot g \Rightarrow$$

$$\Rightarrow v = \sqrt{\mu \cdot g \cdot R} = \sqrt{0.8 \cdot 10 \cdot 50} = 20 \frac{\text{m}}{\text{s}} = \boxed{72 \frac{\text{km}}{\text{h}}}$$

* Velocidad de volcado:



Tomando momentos respecto a O:

$$m \cdot g \cdot \frac{a}{2} - \frac{m \cdot v^2}{R} \cdot \frac{b}{2} = 0 \Rightarrow 10 \cdot \frac{0.6}{2} - \frac{v^2}{50} \cdot \frac{0.8}{2} = 0 \Rightarrow$$

$$\Rightarrow 3 - 8 \cdot 10^{-3} \cdot v^2 = 0 \Rightarrow v = 19.4 \frac{\text{m}}{\text{s}} = \boxed{69.7 \frac{\text{km}}{\text{h}}} \quad (57)$$

29) $m = 2 \text{ kg}$

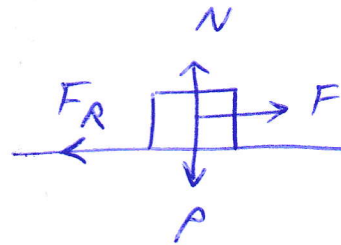
$v_0 = 0$

$\mu = 0.3$

d'F?

$v = 20 \frac{\text{km}}{\text{h}} = 5.56 \frac{\text{m}}{\text{s}}$

$t = 3 \text{ s}$



$F - F_R = m \cdot a$

$a = \frac{\Delta v}{\Delta t} = \frac{5.56 - 0}{3 - 0} = 1.85 \frac{\text{m}}{\text{s}^2}$

$F_R = \mu \cdot m \cdot g = 0.3 \cdot 2 \cdot 10 = 6 \text{ N}$

$F = F_R + m \cdot a = 6 + 2 \cdot 1.85 = \boxed{9.7 \text{ N}}$

30) $m = 100 \text{ kg}$

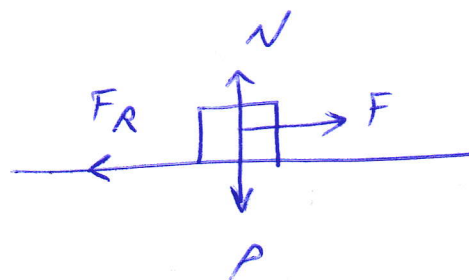
$v_0 = 0$

d'F?

$e = 50 \text{ m}$

$t = 6 \text{ s}$

$\mu = 0.4$



$F_R = \mu \cdot m \cdot g = 0.4 \cdot 100 \cdot 10 = 400 \text{ N}$

$F - F_R = m \cdot a$

$e = \frac{1}{2} \cdot a \cdot t^2 \Rightarrow a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 50}{6^2} = \frac{100}{36} = 2.78 \frac{\text{m}}{\text{s}^2}$

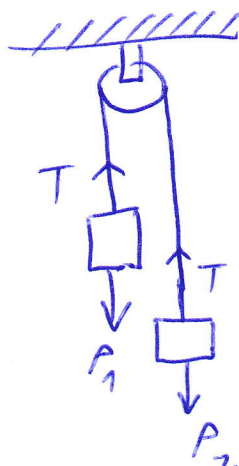
$F = F_R + m \cdot a = 400 + 100 \cdot 2.78 = \boxed{678 \text{ N}}$

31 $m_1 = 1.5 \text{ kg}$

$m_2 = 2 \text{ kg}$

a) $\vec{a}$?

b) $\vec{T}$?



$$\sum \vec{F} = 0 \Rightarrow P_2 - P_1 = m_T \cdot a \Rightarrow a = \frac{P_2 - P_1}{m_1 + m_2} =$$

$$= \frac{20 - 15}{1.5 + 2} = \frac{5}{3.5} = \boxed{1.43 \frac{\text{m}}{\text{s}^2}}$$

$$P_2 - T = m_2 \cdot a \Rightarrow T = P_2 - m_2 \cdot a = 20 - 2 \cdot 1.43 = \boxed{17.1 \text{ N}}$$

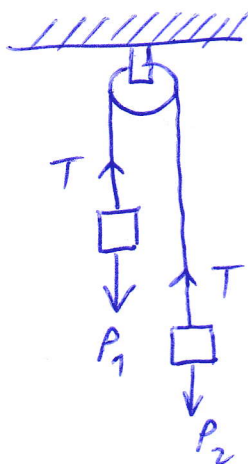
32 $m_1 = 20 \text{ kg}$

$m_2 = 30 \text{ kg}$

$\vec{a}$?

$\vec{e}$?

$t = 3 \text{ s}$



$$\sum \vec{F} = 0 \Rightarrow P_2 - P_1 = m_T \cdot a \Rightarrow a = \frac{P_2 - P_1}{m_T} =$$

$$= \frac{300 - 200}{20 + 30} = \frac{100}{50} = 2 \frac{\text{m}}{\text{s}^2}$$

$$T - P_1 = m_1 \cdot a \Rightarrow T = P_1 + m_1 \cdot a = 200 + 20 \cdot 2 = \boxed{240 \text{ N}}$$

$$e = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot 2 \cdot 3^2 = 9 \text{ m}$$

El espacio total será el doble del que recorre cada masa:
(59) $e_T = \boxed{18 \text{ m}}$

33) $m = 1000 \text{ kg}$

$F = 35 \text{ N}$

¿N?

$$F_1 = \frac{F_2}{2 \cdot N} \Rightarrow$$

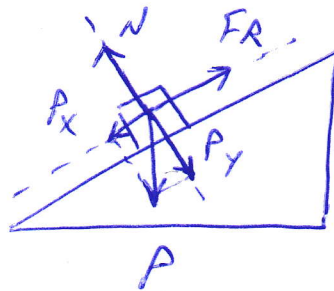
$$\Rightarrow N = \frac{F_2}{2 \cdot F_1} = \frac{1000 \cdot 10}{2 \cdot 35} =$$

$$= \frac{10000}{70} = \boxed{143}$$

34) $\alpha = 60^\circ$

¿a?

$\mu = 0.24$



$$\sum \vec{F} = m \cdot a \Rightarrow P_x - F_R = m \cdot a \Rightarrow$$

$$\Rightarrow m \cdot g \cdot \sin \alpha - \mu \cdot m \cdot g \cdot \cos \alpha = m \cdot a \Rightarrow$$

$$\Rightarrow a = g \cdot (\sin \alpha - \mu \cdot \cos \alpha) =$$

$$= 10 \cdot (\sin 60^\circ - 0.24 \cdot \cos 60^\circ) =$$

$$= \boxed{7.46 \frac{\text{m}}{\text{s}^2}}$$

35) $m = 70 \text{ kg}$

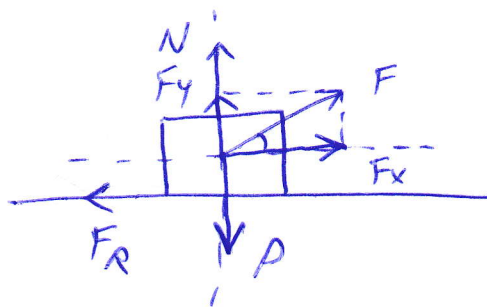
$v_0 = 0$

$\mu = 0.35$

$\alpha = 60^\circ$

$v = 80 \frac{\text{km}}{\text{h}} = 22.2 \frac{\text{m}}{\text{s}}$

$t = 10 \text{ s}$



$$\begin{aligned} N + F_y &= P \Rightarrow N = P - F_y = \\ &= m \cdot g - F \cdot \sin \alpha = \\ &= 70 \cdot 10 - F \cdot \sin 60^\circ = \\ &= 700 - 0.866 \cdot F \end{aligned}$$

$$F_R = \mu \cdot N = 0.35 \cdot (700 - 0.866 \cdot F) = 245 - 0.303 \cdot F$$

$$F_x = F \cdot \cos \alpha = F \cdot \cos 60^\circ = 0.5 \cdot F ; \quad a = \frac{\Delta v}{\Delta t} = \frac{22.2}{10} = 2.22 \frac{\text{m}}{\text{s}^2}$$

$$F_x - F_R = m \cdot a$$

$$0.5 \cdot F - 245 + 0.303 \cdot F = 70 \cdot 2.22$$

$$0.5 \cdot F + 0.303 \cdot F = 70 \cdot 2.22 + 245$$

$$F = \frac{70 \cdot 2.22 + 245}{0.5 + 0.303} = \boxed{499 \text{ N}}$$

36) $m = 90 \text{ kg}$

$v_0 = 0$

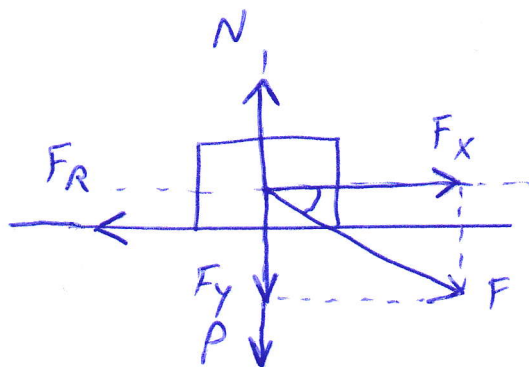
$\mu = 0.4$

$\delta T?$

$e = 100 \text{ m}$

$t = 12 \text{ s}$

$\alpha = 35^\circ$



$$\sum \vec{F} = m \cdot \vec{a} \Rightarrow F_x - F_R = m \cdot a$$

$$e = \frac{1}{2} \cdot a \cdot t^2 \Rightarrow a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 100}{12^2} = 1.39 \frac{\text{m}}{\text{s}^2}$$

$$F_x = F \cdot \cos \alpha = F \cdot \cos 35^\circ = 0.819 \cdot F$$

$$N = P + F_y = m \cdot g + F \cdot \sin \alpha = 90 \cdot 10 + F \cdot \sin 35^\circ = 900 + 0.574 \cdot F$$

$$F_R = \mu \cdot N = 0.4 \cdot (900 + 0.574 \cdot F) = 360 + 0.23 \cdot F$$

$$0.819 \cdot F - 360 - 0.23 \cdot F = 90 \cdot 1.39 \Rightarrow F = \boxed{824 \text{ N}}$$

37) $m = 70 \text{ kg}$

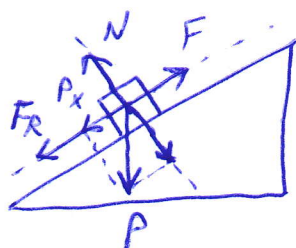
$\alpha = 30^\circ$

$\delta F?$

$e = 10 \text{ m}$

$t = 4 \text{ s}$

$\mu = 0.15$



$$e = \frac{1}{2} \cdot a \cdot t^2$$

$$a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 10}{4^2} = 1.25 \frac{\text{m}}{\text{s}^2}$$

$$F - P_x - F_R = m \cdot a$$

$$P_x = m \cdot g \cdot \sin \alpha = 70 \cdot 10 \cdot \sin 30^\circ = 350 \text{ N}$$

$$F_R = \mu \cdot m \cdot g \cdot \cos \alpha = 0.15 \cdot 70 \cdot 10 \cdot \cos 30^\circ = 90.9 \text{ N}$$

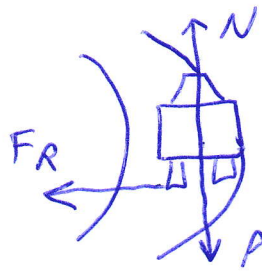
$$F = P_x + F_R + m \cdot a = 350 + 90.9 + 70 \cdot 1.25 =$$

$$= \boxed{528 \text{ N}}$$

38) $R = 50 \text{ m}$

$\mu = 0.35$

$\hat{v} \rightarrow \frac{\text{km}}{\text{h}}$



$F_c = F_R$

$\frac{m \cdot v^2}{R} = \mu \cdot m \cdot g$

$v^2 = \mu \cdot g \cdot R \Rightarrow v = \sqrt{\mu \cdot g \cdot R} = \sqrt{0.35 \cdot 10 \cdot 50} =$

$= 13.2 \frac{\text{m}}{\text{s}} = \boxed{47.6 \frac{\text{km}}{\text{h}}}$

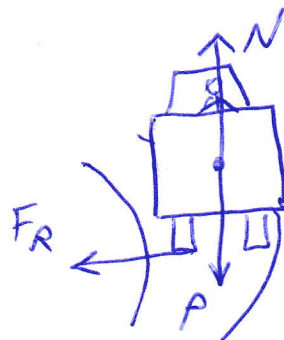
39) $m = 2 \text{ ton}$

$R = 70 \text{ m}$

$v = 80 \frac{\text{km}}{\text{h}} = 22.2 \frac{\text{m}}{\text{s}}$

$\mu = 0.8$

$\hat{a}_c, F_c?$



$F_R = F_c$

$\mu \cdot m \cdot g = \frac{m \cdot v^2}{R}$

$v^2 = \mu \cdot g \cdot R \Rightarrow v = \sqrt{\mu \cdot g \cdot R} =$

$= \sqrt{0.8 \cdot 10 \cdot 70} = 23.7 \frac{\text{m}}{\text{s}}$

$v = 22.2 \frac{\text{m}}{\text{s}} < v_{\text{max}} = 23.7 \frac{\text{m}}{\text{s}} \Rightarrow \boxed{\text{No se saldrá.}}$

$a_c = \frac{v^2}{R} = \frac{22.2^2}{70} = \boxed{7.04 \frac{\text{m}}{\text{s}^2}}$

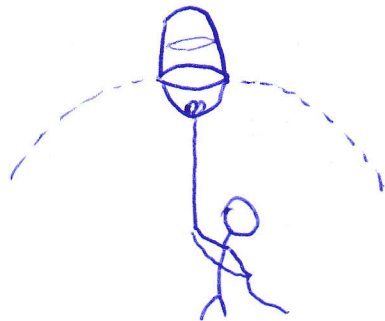
$F_c = m \cdot a_c = 2000 \cdot 7.04 = \boxed{1.41 \cdot 10^4 \text{ J}}$

40 $l = 0.8 \text{ m}$

$\hat{c} T?$

$\hat{c} T_1, T_2?$

$m = 12 \text{ kg}$



* Para la velocidad mínima: $P = F_c \Rightarrow$

$$\Rightarrow m \cdot g = \frac{m \cdot v^2}{l} \Rightarrow v = \sqrt{g \cdot l} = \sqrt{10 \cdot 0.8} = 2.83 \frac{\text{m}}{\text{s}}$$

$$T = \frac{2 \cdot \pi \cdot R}{v} = \frac{2 \cdot \pi \cdot 0.8}{2.83} = \boxed{1.78 \text{ s}}$$

* En el punto más alto: $T_1 + P = F_c$. Al ser: $P = F_c \Rightarrow$

$$\Rightarrow \boxed{T_1 = 0}$$

* En el punto más bajo: $T_2 - P = F_c \Rightarrow$

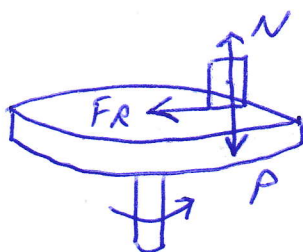
$$\Rightarrow T_2 - m \cdot g = \frac{m \cdot v^2}{l} \Rightarrow T_2 = m \cdot g + \frac{m \cdot v^2}{l} =$$

$$= 12 \cdot 10 + \frac{12 \cdot 2.83^2}{0.8} = \boxed{240 \text{ N}}$$

41 $R = 4 \text{ m}$

$\hat{c} \omega_{\text{máx.}}?$

$\mu = 0.4$



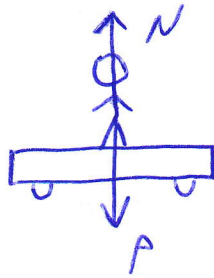
$$F_R = F_c$$

$$\mu \cdot m \cdot g = \frac{m \cdot v^2}{R}$$

$$v = \sqrt{\mu \cdot g \cdot R} = \sqrt{0.4 \cdot 10 \cdot 4} = \sqrt{16} = 4 \frac{\text{m}}{\text{s}}$$

$$v = \omega \cdot R \Rightarrow \omega = \frac{v}{R} = \frac{4}{4} = \boxed{1 \frac{\text{rad}}{\text{s}}}$$

42) $m = 80 \text{ kg}$
 $v = \text{cte}$
 $a = 2 \frac{\text{m}}{\text{s}^2}$



La indicación de la báscula será:
 $\frac{N}{g}$

a) $N = P = m \cdot g = 80 \cdot 10 = 800 \text{ N}$; $\frac{N}{g} = \frac{800}{10} = \boxed{80 \text{ kg}}$

b) $v = \text{cte} \Rightarrow R = 0 \Rightarrow N = P = 800 \text{ N} \Rightarrow \frac{N}{g} = \boxed{80 \text{ kg}}$

c) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = +m \cdot a \cdot \vec{j} \Rightarrow$
 $\Rightarrow N = P + m \cdot a = 800 + 80 \cdot 2 = 800 + 160 = 960 \text{ N}$

$\frac{N}{g} = \frac{960}{10} = \boxed{96 \text{ kg}}$

d) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = -m \cdot a \cdot \vec{j} \Rightarrow$
 $\Rightarrow N - P = -m \cdot a \Rightarrow N = P - m \cdot a = 800 - 80 \cdot 2 = 640 \text{ N}$

$\frac{N}{g} = \frac{640}{10} = \boxed{64 \text{ kg}}$

e) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = -m \cdot a \cdot \vec{j} \Rightarrow$
 $\Rightarrow N - P = -m \cdot a \Rightarrow N = P - m \cdot a = 800 - 160 = 640 \text{ N}$

$\frac{N}{g} = \frac{640}{10} = \boxed{64 \text{ kg}}$

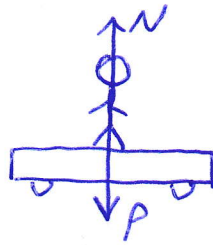
f) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = +m \cdot a \cdot \vec{j} \Rightarrow$
 $\Rightarrow N - P = m \cdot a \Rightarrow N = P + m \cdot a = 800 + 160 = 960 \text{ N}$

$\frac{N}{g} = \frac{960}{10} = \boxed{96 \text{ kg}} \quad (65)$

43) $m = 70 \text{ kg}$

$v = 5 \frac{\text{m}}{\text{s}}$

$a = 2 \frac{\text{m}}{\text{s}^2}$



La indicación de la báscula será:

$$\frac{N}{g}$$

a) $N = P = m \cdot g = 70 \cdot 10 = 700 \text{ N}$; $\frac{N}{g} = \frac{700}{10} = \boxed{70 \text{ kg}}$

b) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = +m \cdot a \cdot \vec{j} \Rightarrow$
 $\Rightarrow N - P = m \cdot a \Rightarrow N = P + m \cdot a = 700 + 70 \cdot 2 = 840 \text{ N}$

$$\frac{N}{g} = \frac{840}{10} = \boxed{84 \text{ kg}}$$

c) $\sum \vec{F} = m \cdot \vec{a} \Rightarrow N \cdot \vec{j} - P \cdot \vec{j} = -m \cdot a \cdot \vec{j} \Rightarrow$

$$\Rightarrow N - P = -m \cdot a \Rightarrow N = P - m \cdot a = 700 - 70 \cdot 2 =$$

$$= 700 - 140 = 560 \text{ N} \Rightarrow \frac{N}{g} = \frac{560}{10} = \boxed{56 \text{ kg}}$$

d) $N = 0 \Rightarrow \frac{N}{g} = \boxed{0}$

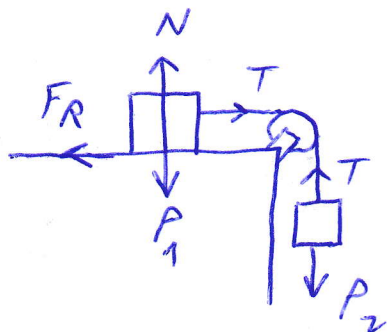
Al caer la báscula con la misma aceleración que la persona, no hay fuerza normal, pues la persona no presiona la superficie de la báscula.

44) $\vec{a}, T?$

$$m_1 = 2 \text{ kg}$$

$$m_2 = 6 \text{ kg}$$

$$\mu = 0.2$$



$$\sum \vec{F} = m \cdot \vec{a}$$

$$P_2 - F_R = m_T \cdot a$$

$$F_R = \mu \cdot m_1 \cdot g =$$

$$= 0.2 \cdot 2 \cdot 10 = 4 \text{ N}$$

$$a = \frac{P_2 - F_R}{m_T} = \frac{60 - 4}{2 + 6} = \frac{56}{8} = \boxed{7 \frac{\text{m}}{\text{s}^2}}$$

$$T - F_R = m_1 \cdot a \Rightarrow T = F_R + m_1 \cdot a = 4 + 2 \cdot 7 = 4 + 14 = \boxed{18 \text{ N}}$$

45) $m_1 = 2 \text{ kg}$

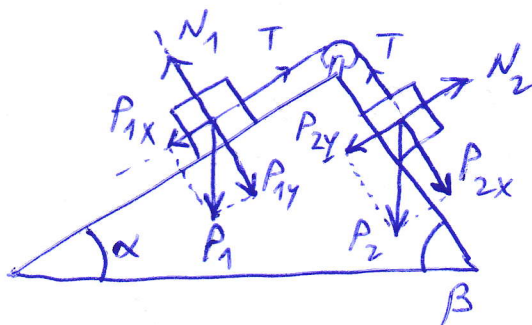
$$m_2 = 6 \text{ kg}$$

$$\mu = 0.2$$

$$\alpha = 30^\circ$$

$$\beta = 60^\circ$$

$\vec{a}, T?$



No se han dibujado F_{R1} ni F_{R2} .

$$\sum \vec{F} = m \cdot \vec{a} \Rightarrow P_{2x} - P_{1x} - F_{R1} - F_{R2} = m_T \cdot a$$

$$P_{1x} = m_1 \cdot g \cdot \sin \alpha = 2 \cdot 10 \cdot \sin 30^\circ = 10 \text{ N}$$

$$P_{2x} = m_2 \cdot g \cdot \sin \beta = 6 \cdot 10 \cdot \sin 60^\circ = 52 \text{ N}$$

$$F_{R1} = \mu \cdot m_1 \cdot g \cdot \cos \alpha = 0.2 \cdot 2 \cdot 10 \cdot \cos 30^\circ = 3.46 \text{ N}$$

$$F_{R2} = \mu \cdot m_2 \cdot g \cdot \cos \beta = 0.2 \cdot 6 \cdot 10 \cdot \cos 60^\circ = 6 \text{ N}$$

$$a = \frac{P_{2x} - P_{1x} - F_{R1} - F_{R2}}{m_1 + m_2} = \frac{52 - 10 - 3.46 - 6}{2 + 6} = \boxed{4.07 \frac{\text{m}}{\text{s}^2}}$$

$$T - P_{1x} - F_{R1} = m_1 \cdot a \Rightarrow T = P_{1x} + F_{R1} + m_1 \cdot a =$$

$$= 10 + 3.46 + 2 \cdot 4.07 = \boxed{21.6 \text{ N}}$$

(67)

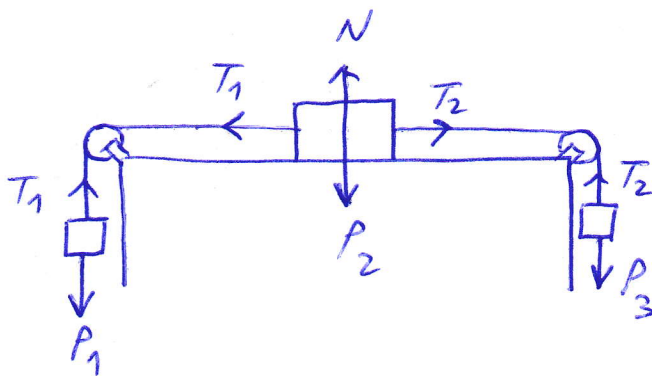
46) $m_1 = 2 \text{ kg}$

$m_2 = 6 \text{ kg}$

$m_3 = 7 \text{ kg}$

$\mu = 0.2$

$\vec{a}, T_1, T_2?$



$P_1 = m_1 \cdot g = 2 \cdot 10 = 20 \text{ N}$

$P_2 = m_2 \cdot g = 6 \cdot 10 = 60 \text{ N}; P_3 = m_3 \cdot g = 7 \cdot 10 = 70 \text{ N}$

$F_R = \mu \cdot P_2 = 0.2 \cdot 60 = 12 \text{ N}$

$\sum \vec{F} = m \cdot \vec{a} \Rightarrow P_3 - P_1 - F_R = m_T \cdot a \Rightarrow$

$\Rightarrow a = \frac{P_3 - P_1 - F_R}{m_T} = \frac{70 - 20 - 12}{2 + 6 + 7} = \frac{38}{15} = \boxed{2.53 \frac{\text{m}}{\text{s}^2}}$

$T_1 - P_1 = m_1 \cdot a \Rightarrow T_1 = P_1 + m_1 \cdot a = 20 + 2 \cdot 2.53 = \boxed{25.1 \text{ N}}$

$P_3 - T_2 = m_3 \cdot a \Rightarrow T_2 = P_3 - m_3 \cdot a = 70 - 7 \cdot 2.53 = \boxed{52.3 \text{ N}}$

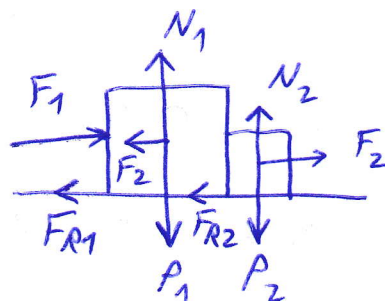
47) $m_1 = 8 \text{ kg}$

$m_2 = 4 \text{ kg}$

$\vec{a}, F_2?$

a) $F_1 = 40 \text{ N}, \mu = 0$

b) $F_1 = 40 \text{ N}, \mu = 0.33$



a) $F_{R1} = \mu \cdot m_1 \cdot g = 0; F_{R2} = 0$

Al sistema completo: $F_1 = m_T \cdot a \Rightarrow$
 $\Rightarrow a = \frac{F_1}{m_T} = \frac{40}{12} = \boxed{3.33 \frac{\text{m}}{\text{s}^2}}$

Para el primer bloque: $F_1 - F_2 - F_{R1} = m_1 \cdot a \Rightarrow$

$\Rightarrow F_2 = F_1 - F_{R1} - m_1 \cdot a = 40 - 0 - 8 \cdot 3.33 = \boxed{13.4 \text{ N}}$

b) $F_{R1} = \mu \cdot m_1 \cdot g = 0.33 \cdot 8 \cdot 10 = 26.4 \text{ N}; F_{R2} = \mu \cdot m_2 \cdot g = 0.33 \cdot 4 \cdot 10 = 13.2 \text{ N};$

$F_1 - F_{R1} - F_{R2} = m_T \cdot a \Rightarrow 40 - 26.4 - 13.2 = 12 \cdot a \Rightarrow a = \boxed{0.0333 \frac{\text{m}}{\text{s}^2}}$

$F_1 - F_2 - F_{R1} = m_1 \cdot a \Rightarrow 40 - F_2 - 26.4 = 8 \cdot 0.0333 \Rightarrow F_2 = \boxed{13.3 \text{ N}}$

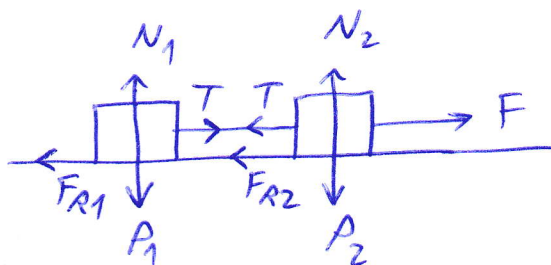
48) $F = 12 \text{ N}$

$\mu = 0.2$

$\hat{a}, T?$

$m_1 = 2 \text{ kg}$

$m_2 = 1 \text{ kg}$



$$F_{R1} = \mu \cdot m_1 \cdot g = 0.2 \cdot 2 \cdot 10 = 4 \text{ N}$$

$$F_{R2} = \mu \cdot m_2 \cdot g = 0.2 \cdot 1 \cdot 10 = 2 \text{ N}$$

$$\Sigma \vec{F} = m \cdot \vec{a} \Rightarrow F - F_{R1} - F_{R2} = m_T \cdot a \Rightarrow$$

$$\Rightarrow a = \frac{F - F_{R1} - F_{R2}}{m_T} = \frac{12 - 4 - 2}{3} = \frac{6}{3} = \boxed{2 \frac{\text{m}}{\text{s}^2}}$$

$$T - F_{R1} = m_1 \cdot a \Rightarrow T = F_{R1} + m_1 \cdot a = 4 + 2 \cdot 2 = \boxed{8 \text{ N}}$$

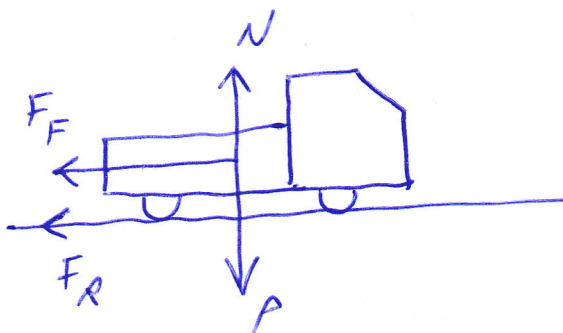
49) $m = 5 \text{ ton}$

$$v = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$$

$\mu = 0.2$

$\hat{F}_F?$

$t = 10 \text{ s}$



$$a = \frac{\Delta v}{\Delta t} = \frac{27.8}{10} = 2.78 \frac{\text{m}}{\text{s}^2}$$

$$F_R = \mu \cdot m \cdot g = 0.2 \cdot 5000 \cdot 10 = 10000 \text{ N}$$

$$\Sigma \vec{F} = m \cdot \vec{a} \Rightarrow F_F + F_R = m \cdot a \Rightarrow$$

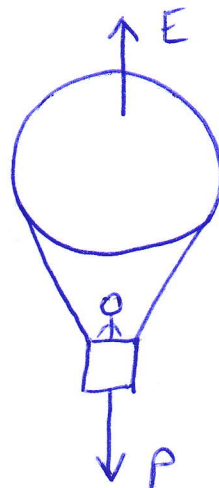
$$\Rightarrow F_F = m \cdot a - F_R = 5000 \cdot 2.78 - 10000 = \boxed{3900 \text{ N}}$$

(50) $m = 180 \text{ kg}$

$a = 0.2 \frac{\text{m}}{\text{s}^2}$

¿m?

$a = 0.5 \frac{\text{m}}{\text{s}^2}$



* Cuando desciende: $\sum \vec{F} = m \cdot \vec{a} \Rightarrow$

$\Rightarrow E \cdot \vec{j} - P \cdot \vec{j} = -m \cdot a \cdot \vec{j} \Rightarrow E - P = -m \cdot a \Rightarrow$

$\Rightarrow E = P - m \cdot a = 180 \cdot 10 - 180 \cdot 0.2 = 1800 - 36 = 1764 \text{ N}$

* Cuando sube: $\sum \vec{F} = m \cdot \vec{a} \Rightarrow$

$\Rightarrow E \cdot \vec{j} - P \cdot \vec{j} = +m \cdot a \cdot \vec{j} \Rightarrow E - P = m \cdot a \Rightarrow$

$\Rightarrow 1764 - m \cdot 10 = m \cdot 0.5 \Rightarrow m = \frac{1764}{10.5} = 168 \text{ kg}$

* Masa de lastre:

$m = 180 - 168 = 12 \text{ kg}$

(51) ¿F?

$m = 80 \text{ kg}$

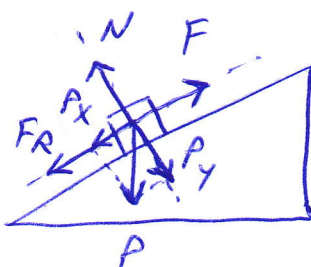
$h = 3 \text{ m}$

$t = 4 \text{ s}$

$v_0 = 0$

$\mu = 0.25$

$\alpha = 35^\circ$



$\sum \vec{F} = m \cdot \vec{a} \Rightarrow$

$\Rightarrow F - P_x - F_R = m \cdot a$

$e = \frac{1}{2} \cdot a \cdot t^2$

$\sin \alpha = \frac{h}{e} \Rightarrow e = \frac{h}{\sin \alpha} = \frac{3}{\sin 35^\circ} = 5.23 \text{ m}$

$a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 5.23}{4^2} = 0.654 \frac{\text{m}}{\text{s}^2}$

$P_x = m \cdot g \cdot \sin \alpha = 80 \cdot 10 \cdot \sin 35^\circ = 459 \text{ N}$

$F_R = \mu \cdot m \cdot g \cdot \cos \alpha = 0.25 \cdot 80 \cdot 10 \cdot \cos 35^\circ = 164 \text{ N}$

$F = P_x + F_R + m \cdot a = 459 + 164 + 80 \cdot 0.654 = 675 \text{ N}$

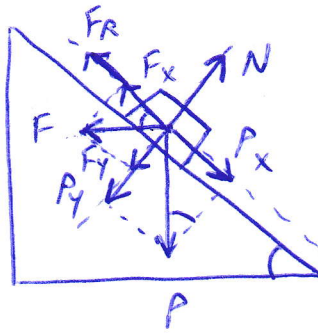
(52) $\hat{c} F?$

$$m = 80 \text{ kg}$$

$$v = cte$$

$$\mu = 0.25$$

$$\alpha = 28^\circ$$



$$v = cte \Rightarrow$$

$$\Rightarrow R = 0 \Rightarrow$$

$$\Rightarrow F_R + F_x = P_x$$

$$P_x = m \cdot g \cdot \sin \alpha = 80 \cdot 10 \cdot \sin 28^\circ = 376 \text{ N}$$

$$F_x = F \cdot \cos \alpha = F \cdot \cos 28^\circ = 0.883 \cdot F$$

$$N = P_y + F_y = m \cdot g \cdot \cos \alpha + F \cdot \sin \alpha =$$

$$= 80 \cdot 10 \cdot \cos 28^\circ + F \cdot \sin 28^\circ = 706 + 0.47 \cdot F$$

$$F_R = \mu \cdot N = 0.25 \cdot (706 + 0.47 \cdot F) = 176 + 0.117 \cdot F$$

$$F_R + F_x = P_x \Rightarrow 176 + 0.117 \cdot F + 0.883 \cdot F = 376 \Rightarrow$$

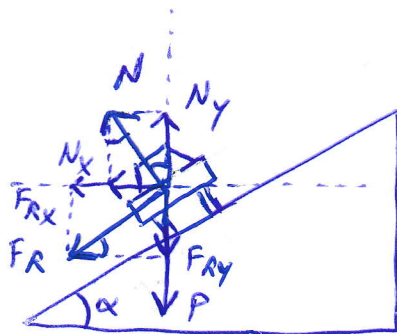
$$\Rightarrow F = \boxed{200 \text{ N}}$$

(53) $\alpha = 60^\circ$

$$R = 50 \text{ m}$$

$$v = 150 \frac{\text{km}}{\text{h}} = 41.7 \frac{\text{m}}{\text{s}}$$

$$\mu = 0.3$$



$$\left. \begin{aligned} N_x + F_{Rx} &= F_c \\ N_y - F_{Ry} &= P \end{aligned} \right\}$$

$$\left. \begin{aligned} N \cdot \sin \alpha + \mu \cdot N \cdot \cos \alpha &= \frac{m \cdot v^2}{R} \\ N \cdot \cos \alpha - \mu \cdot N \cdot \sin \alpha &= m \cdot g \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{aligned} N \cdot (\sin \alpha + \mu \cdot \cos \alpha) &= \frac{m \cdot v^2}{R} \\ N \cdot (\cos \alpha - \mu \cdot \sin \alpha) &= m \cdot g \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \sin \alpha + \mu \cdot \cos \alpha &= \frac{v^2}{R \cdot g} \\ \cos \alpha - \mu \cdot \sin \alpha &= \frac{m \cdot g}{N} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow v_{\max} = \sqrt{R \cdot g \cdot \frac{\sin \alpha + \mu \cdot \cos \alpha}{\cos \alpha - \mu \cdot \sin \alpha}} = \sqrt{50 \cdot 10 \cdot \frac{\sin 60^\circ + 0.3 \cdot \cos 60^\circ}{\cos 60^\circ - 0.3 \cdot \sin 60^\circ}} =$$

$$= 46.1 \frac{\text{m}}{\text{s}} ; v < v_{\max} \Rightarrow \boxed{\text{No se sale}}$$

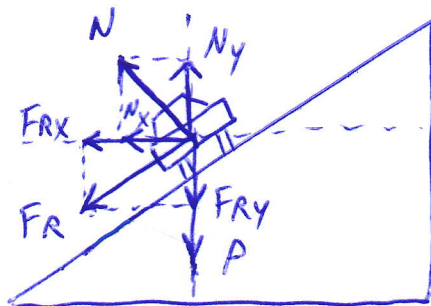
(71)

54) $\vec{v} ? \rightarrow \frac{km}{h}$

$\alpha = 30^\circ$

$R = 60 m$

$\mu = 0.3$



$$\left. \begin{aligned} N_x + F_{Rx} &= F_c \\ N_y - F_{Ry} &= P \end{aligned} \right\}$$

$$\left. \begin{aligned} N \cdot \sin \alpha + \mu \cdot N \cdot \cos \alpha &= \frac{m \cdot v^2}{R} \\ N \cdot \cos \alpha - \mu \cdot N \cdot \sin \alpha &= m \cdot g \end{aligned} \right\} \begin{aligned} N \cdot (\sin \alpha + \mu \cdot \cos \alpha) &= \frac{m \cdot v^2}{R} \\ N \cdot (\cos \alpha - \mu \cdot \sin \alpha) &= m \cdot g \end{aligned}$$

$$\frac{\sin \alpha + \mu \cdot \cos \alpha}{\cos \alpha - \mu \cdot \sin \alpha} = \frac{v^2}{R \cdot g} ; \quad v = \sqrt{R \cdot g \cdot \frac{\sin \alpha + \mu \cdot \cos \alpha}{\cos \alpha - \mu \cdot \sin \alpha}} =$$

$$= \sqrt{60 \cdot 10 \cdot \frac{\sin 30^\circ + 0.3 \cdot \cos 30^\circ}{\cos 30^\circ - 0.3 \cdot \sin 30^\circ}} = 25.2 \frac{m}{s} = \boxed{90.8 \frac{km}{h}}$$

55) $m = 3 kg$

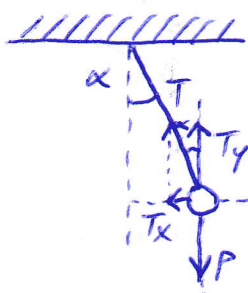
$l = 5 m$

$\alpha = 10^\circ$

a) $\vec{v} T, A?$

b) $\vec{v} T, R?$

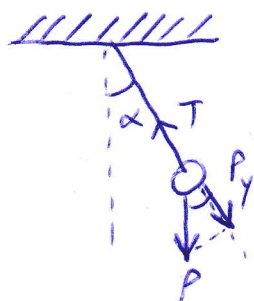
$g = 9.8 \frac{m}{s^2}$



$$\sin \alpha = \frac{A/2}{l}$$

$$A = 2 \cdot l \cdot \sin \alpha = 2 \cdot 5 \cdot \sin 10^\circ = \boxed{1.74 m}$$

$$T = 2 \cdot \pi \cdot \sqrt{\frac{l}{g}} = 2 \cdot \pi \cdot \sqrt{\frac{5}{10}} = \boxed{4.44 s}$$



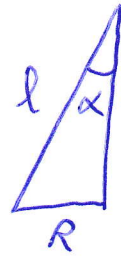
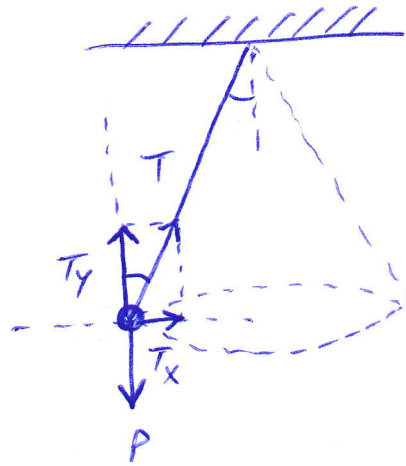
en el extremo: $T = P_y = m \cdot g \cdot \cos \alpha =$
 $= 3 \cdot 10 \cdot \cos 10^\circ = \boxed{29.5 N}$

(56) $m = 2 \text{ kg}$

$l = 1/2 \text{ m}$

$\alpha = 40^\circ$

$\vec{P}, T?$



$$\left. \begin{array}{l} T_y = P \\ T_x = F_c \end{array} \right\} \left. \begin{array}{l} T \cdot \cos \alpha = m \cdot g \\ T \cdot \sin \alpha = \frac{m \cdot v^2}{R} \end{array} \right\} \quad T = \frac{m \cdot g}{\cos \alpha} = \frac{2 \cdot 10}{\cos 40^\circ} = \boxed{26.1 \text{ N}}$$

$$\sin \alpha = \frac{R}{l} \Rightarrow R = l \cdot \sin \alpha = 1/2 \cdot \sin 40^\circ = 0.771 \text{ m}$$

$$v = \sqrt{\frac{R \cdot T \cdot \sin \alpha}{m}} = \sqrt{\frac{0.771 \cdot 26.1 \cdot \sin 40^\circ}{2}} = 2.54 \frac{\text{m}}{\text{s}}$$

$$T = \frac{2 \cdot \pi \cdot R}{v} = \frac{2 \cdot \pi \cdot 0.771}{2.54} = \boxed{1.91 \text{ s}}$$

(57) $m = 0.4 \text{ kg}$

$\alpha = 60^\circ$

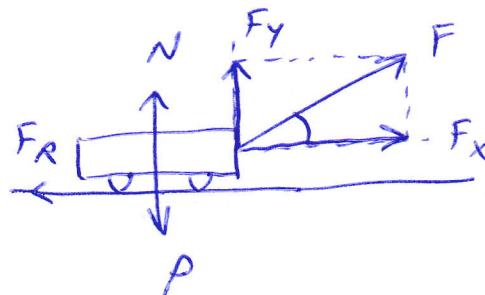
$\mu = 0.38$

a) $v = \text{cte}$

b) $e = 5 \text{ m}$

$t = 2 \text{ s}$

$v_0 = 0$



$$F_R = \mu \cdot N$$

$$F_R = 0.38 \cdot (4 - 0.866 \cdot F) = 1.52 - 0.329 \cdot F$$

a) $v = \text{cte} \Rightarrow F_x = F_R \Rightarrow F \cdot \cos 60^\circ = 1.52 - 0.329 \cdot F \Rightarrow$

$$\Rightarrow F = \boxed{1.83 \text{ N}}$$

$$N + F_y = P$$

$$N = P - F_y =$$

$$= m \cdot g - F \cdot \sin \alpha =$$

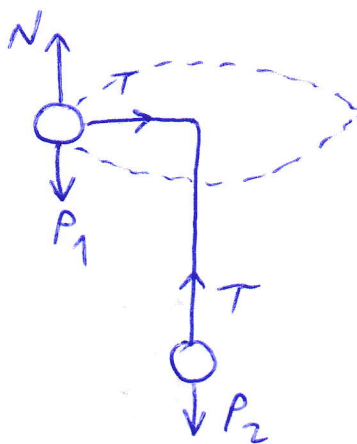
$$= 0.4 \cdot 10 - F \cdot \sin 60^\circ =$$

$$= 4 - 0.866 \cdot F$$

b) $e = \frac{1}{2} \cdot a \cdot t^2 \Rightarrow a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 5}{2^2} = 2.5 \frac{\text{m}}{\text{s}^2}$

$$F_x - F_R = m \cdot a \Rightarrow F \cdot \cos 60^\circ - 1.52 + 0.329 \cdot F = 0.4 \cdot 2.5 \Rightarrow F = \boxed{3.04 \text{ N}}$$

58) $m_1 = 0.3 \text{ kg}$
 $m_2 = 0.5 \text{ kg}$
 $\dot{\theta}, \omega?$
 $R = 0.3 \text{ m}$



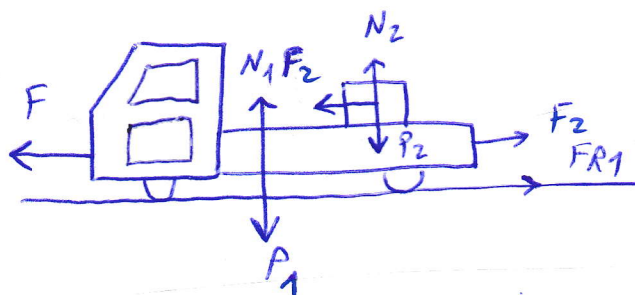
$$\left. \begin{aligned} T &= P_2 \\ T &= F_c = \frac{m_1 v^2}{R} \end{aligned} \right\}$$

$$T = P_2 = m_2 \cdot g = 0.5 \cdot 10 = \boxed{5 \text{ N}}$$

$$v = \sqrt{\frac{T \cdot R}{m_1}} = \sqrt{\frac{5 \cdot 0.3}{0.3}} = 2.24 \frac{\text{m}}{\text{s}}$$

$$v = \omega \cdot R \Rightarrow \omega = \frac{v}{R} = \frac{2.24}{0.3} = \boxed{7.47 \frac{\text{rad}}{\text{s}^2}}$$

59) $m_1 = 2000 \text{ kg}$
 $m_2 = 20 \text{ kg}$
 $\mu = 0.7$



$$\mu = 0.5$$

$$\dot{\theta} F_R, a_2?$$

$$\begin{aligned} a) \quad F_{R1} &= \mu \cdot (m_1 + m_2) \cdot g = \\ &= 0.5 \cdot (2000 + 20) \cdot 10 = 10100 \text{ N} \end{aligned}$$

$$F - F_{R1} - F_2 = m_1 \cdot a \Rightarrow$$

$$\Rightarrow 15000 - 10100 - 20 \cdot a = 2000 \cdot a \Rightarrow a = \boxed{2.43 \frac{\text{m}}{\text{s}^2}}$$

$$F_2 = m_2 \cdot a = 20 \cdot 2.43 = \boxed{486 \text{ N}}$$

b) El paquete se mueve con respecto al camión a

$$\text{partir de: } F_2 = F_{R2} \Rightarrow \mu_2 \cdot a_2 = \mu \cdot m_2 \cdot g \Rightarrow$$

$$\Rightarrow a_2 = \mu \cdot g = 0.7 \cdot 10 = \boxed{7 \frac{\text{m}}{\text{s}^2}}$$

$$F_2 = F_{R2} = m_2 \cdot a = 20 \cdot 7 = \boxed{140 \text{ N}}$$

60 $\mu = 0.3$

$t = 2 \text{ s}$

$e = 5 \text{ m}$

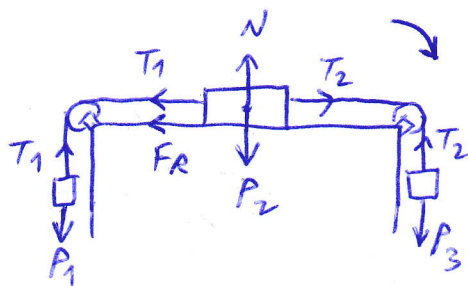
$v_0 = 0$

$a, m_2?$

$m_1 = 3 \text{ kg}$

$m_3 = 12 \text{ kg}$

$T_1, T_2?$



$e = \frac{1}{2} \cdot a \cdot t^2$

$a = \frac{2 \cdot e}{t^2} = \frac{2 \cdot 5}{2^2} = 2.5 \frac{\text{m}}{\text{s}^2}$

$\sum \vec{F} = m \cdot \vec{a} \Rightarrow P_3 - P_1 - F_R = m_T \cdot a$

$F_R = \mu \cdot m_2 \cdot g = 0.3 \cdot m_2 \cdot 10 = 3 \cdot m_2$

$120 - 30 - 3 \cdot m_2 = (3 + m_2 + 12) \cdot 2.5$

$90 - 3 \cdot m_2 = 37.5 + 2.5 \cdot m_2 \Rightarrow m_2 = \boxed{9.55 \text{ kg}}$

$T_1 - P_1 = m_1 \cdot a \Rightarrow T_1 = P_1 + m_1 \cdot a = 30 + 3 \cdot 2.5 = \boxed{37.5 \text{ N}}$

$P_3 - T_2 = m_3 \cdot a \Rightarrow T_2 = P_3 - m_3 \cdot a = 120 - 12 \cdot 2.5 = \boxed{90 \text{ N}}$

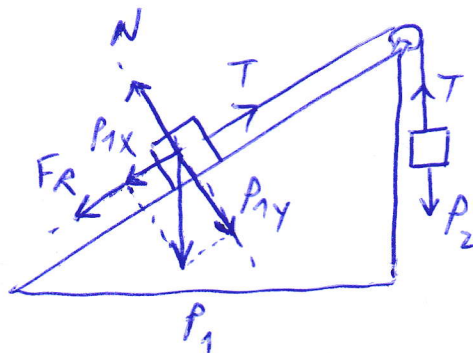
61 $a, T?$

$m_1 = 2 \text{ kg}$

$m_2 = 6 \text{ kg}$

$\alpha = 37^\circ$

$\mu = 0.28$



$\sum \vec{F} = m \cdot \vec{a}$

$P_2 - P_{1x} - F_R = m_T \cdot a$

$P_2 = m_2 \cdot g = 6 \cdot 10 = 60 \text{ N}$

$P_{1x} = m_1 \cdot g \cdot \sin \alpha = 2 \cdot 10 \cdot \sin 37^\circ = 12 \text{ N}$

$F_R = \mu \cdot m_1 \cdot g \cdot \cos \alpha = 0.28 \cdot 2 \cdot 10 \cdot \cos 37^\circ = 4.47 \text{ N}$

$a = \frac{P_2 - P_{1x} - F_R}{m_T} = \frac{60 - 12 - 4.47}{2 + 6} = \frac{43.53}{8} = \boxed{5.44 \frac{\text{m}}{\text{s}^2}}$

$P_2 - T = m_2 \cdot a \Rightarrow T = P_2 - m_2 \cdot a = 60 - 6 \cdot 5.44 =$

$= \boxed{27.4 \text{ N}}$

62) $m_2 = 20 \text{ kg}$

$m_1 = 90 \text{ kg}$

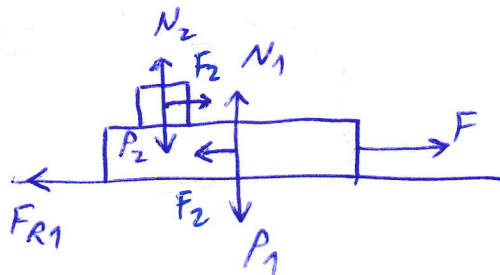
$\mu = 0.4$

a) $\vec{F}_{\text{máx.}}$?

b) $\vec{a}$?

c) $\vec{a}_1, \vec{a}_2$?

$F = 900 \text{ N}$



$$F_{R1} = \mu \cdot (m_1 + m_2) \cdot g = 0.4 \cdot 110 \cdot 10 = 440 \text{ N}$$

$$F_{R2} = \mu \cdot m_2 \cdot g = 0.4 \cdot 20 \cdot 10 = 80 \text{ N}$$

a) $\sum \vec{F} = 0 \Rightarrow F - F_{R1} - F_{R2} + F_{R2} = 0 \Rightarrow$

$$\Rightarrow F = F_{R1} = \boxed{440 \text{ N}}$$

b) Para el de arriba: $\sum \vec{F} = m \cdot \vec{a} \Rightarrow F_{R2} = m \cdot a \Rightarrow$

$$\Rightarrow \mu \cdot m \cdot g = m \cdot a \Rightarrow a = \mu \cdot g = 0.4 \cdot 10 = \boxed{4 \frac{\text{m}}{\text{s}^2}}$$

A partir de $4 \frac{\text{m}}{\text{s}^2}$, hay movimiento relativo entre los dos bloques.

c) $F - F_{R1} - F_{R2} = m_1 \cdot a_1$

$$a_1 = \frac{F - F_{R1} - F_{R2}}{m_1} = \frac{900 - 440 - 80}{90} = \frac{380}{90} = \boxed{4.22 \frac{\text{m}}{\text{s}^2}}$$

$a_2 = \boxed{4 \frac{\text{m}}{\text{s}^2}}$ pues, a partir de $4 \frac{\text{m}}{\text{s}^2}$, el de arriba

se mueve con respecto al de abajo.

63) $m_2 = 5 \text{ kg}$

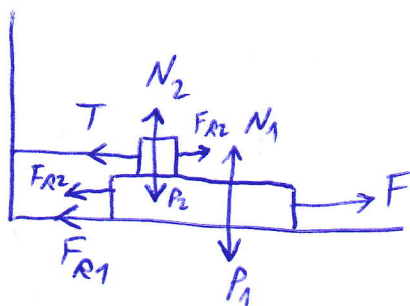
$m_1 = 20 \text{ kg}$

$\mu = 0.3$

a) $d' F_{mín}$?

b) $d' a, T$?

$F = 120 \text{ N}$



$$F_{R1} = \mu \cdot (m_1 + m_2) \cdot g = 0.3 \cdot (20 + 5) \cdot 10 = 75 \text{ N}$$

$$F_{R2} = \mu \cdot m_2 \cdot g = 0.3 \cdot 5 \cdot 10 = 15 \text{ N}$$

a) $F_{mín} \Rightarrow A = 0 \Rightarrow F = F_{R1} + F_{R2} = 75 + 15 = \boxed{90 \text{ N}}$

b) Para el cuerpo de abajo: $F - F_{R1} - F_{R2} = m_1 \cdot a \Rightarrow$

$$\Rightarrow a = \frac{F - F_{R1} - F_{R2}}{m_1} = \frac{120 - 75 - 15}{20} = \frac{30}{20} = \boxed{1.5 \frac{\text{m}}{\text{s}^2}}$$

$T = F_{R2} = \boxed{15 \text{ N}}$

64) $m_1 = 60 \text{ kg}$

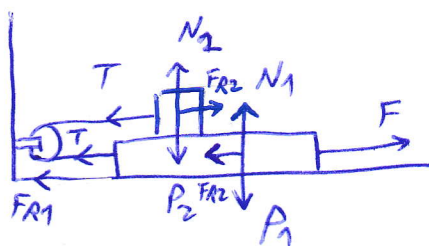
$m_2 = 20 \text{ kg}$

$\mu = 0.4$

a) $d' F_{mín}$?

b) $d' a, T$?

$F = 500 \text{ N}$



$$F_{R2} = \mu \cdot m_2 \cdot g = 0.4 \cdot 20 \cdot 10 = 80 \text{ N}$$

$$F_{R1} = \mu \cdot (m_1 + m_2) \cdot g = 0.4 \cdot (60 + 20) \cdot 10 = 320 \text{ N}$$

a) $F_{mín} \Rightarrow v = \text{cte} \Rightarrow A = 0 \Rightarrow F = F_{R1} + 2 \cdot F_{R2} =$

$$= 320 + 2 \cdot 80 = 320 + 160 = \boxed{480 \text{ N}}$$

b) $F - F_{R1} - 2 \cdot F_{R2} = m_T \cdot a \Rightarrow a = \frac{F - F_{R1} - 2 \cdot F_{R2}}{m_1 + m_2} =$

$$= \frac{500 - 320 - 2 \cdot 80}{60 + 20} = \frac{500 - 480}{80} = \frac{20}{80} = \boxed{0.25 \frac{\text{m}}{\text{s}^2}}$$

$T - F_{R2} = m_2 \cdot a \Rightarrow T = F_{R2} + m_2 \cdot a = 80 + 20 \cdot 0.25 = \boxed{85 \text{ N}}$
(77)

65) $m = 2 \text{ kg}$

$\vec{F}_1 = 3\vec{i} - 8\vec{j}$

$\vec{F}_2 = 7\vec{i} + 4\vec{j}$

ca?

$\sum \vec{F} = m \cdot \vec{a}$

$\sum \vec{F} = (3+7)\vec{i} + (-8+4)\vec{j} = 10\vec{i} - 4\vec{j}$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{10\vec{i} - 4\vec{j}}{2} = 5\vec{i} - 2\vec{j}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29} = \boxed{5.39 \frac{m}{s^2}}$

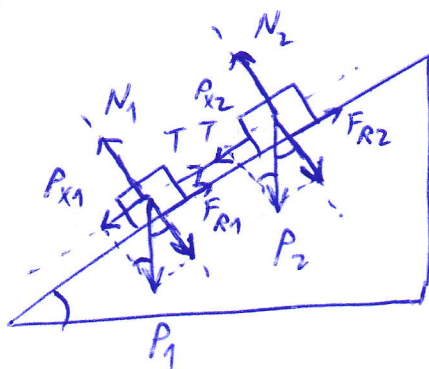
66) $m_1 = 70 \text{ kg}$

$m_2 = 30 \text{ kg}$

$\mu = 0.45$

$\alpha = 50^\circ$

ca, T?



$\sum \vec{F} = m \cdot \vec{a}$

$P_{x1} + P_{x2} - F_{R1} - F_{R2} = m_T \cdot a$

$P_{x1} = m_1 \cdot g \cdot \sin \alpha = 70 \cdot 10 \cdot \sin 50^\circ = 536 \text{ N}$

$P_{x2} = m_2 \cdot g \cdot \sin 50^\circ = 30 \cdot 10 \cdot \sin 50^\circ = 230 \text{ N}$

$F_{R1} = \mu_1 \cdot m_1 \cdot g \cdot \cos \alpha = 0.35 \cdot 70 \cdot 10 \cdot \cos 50^\circ = 158 \text{ N}$

$F_{R2} = \mu_2 \cdot m_2 \cdot g \cdot \cos \alpha = 0.45 \cdot 30 \cdot 10 \cdot \cos 50^\circ = 86.8 \text{ N}$

$a = \frac{P_{x1} + P_{x2} - F_{R1} - F_{R2}}{m_1 + m_2} = \frac{536 + 230 - 158 - 86.8}{70 + 30} = \boxed{5.21 \frac{m}{s^2}}$

$T + P_{x2} - F_{R2} = m_2 \cdot a \Rightarrow T = m_2 \cdot a - P_{x2} + F_{R2} =$

$= 30 \cdot 5.21 - 230 + 86.8 = \boxed{13.1 \text{ N}}$

67) $m_1 = 40 \text{ kg}$

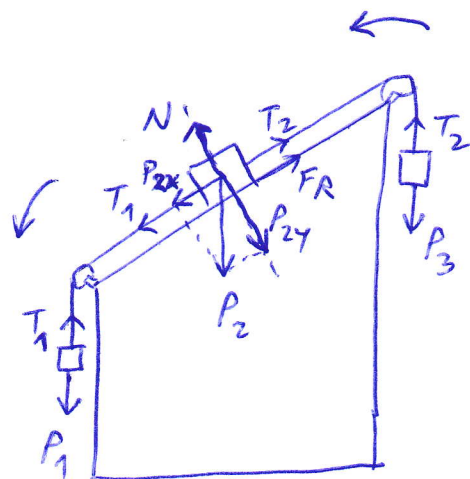
$m_2 = 30 \text{ kg}$

$m_3 = 10 \text{ kg}$

$\mu = 0.45$

$\alpha = 60^\circ$

find a, T_1



$$\sum \vec{F} = m \cdot \vec{a}$$

$$P_1 + P_{2x} - F_R - P_3 = m_T \cdot a$$

$$P_1 = m_1 \cdot g =$$

$$= 40 \cdot 10 = 400 \text{ N}$$

$$P_3 = m_3 \cdot g = 10 \cdot 10 = 100 \text{ N}$$

$$P_{2x} = m_2 \cdot g \cdot \sin \alpha = 30 \cdot 10 \cdot \sin 60^\circ = 260 \text{ N}$$

$$F_R = \mu \cdot m_2 \cdot g \cdot \cos \alpha = 0.45 \cdot 30 \cdot 10 \cdot \cos 60^\circ = 67.5 \text{ N}$$

$$a = \frac{P_1 + P_{2x} - F_R - P_3}{m_T} = \frac{400 + 260 - 67.5 - 100}{40 + 30 + 10} = \frac{492.5}{80} = 6.16 \frac{\text{m}}{\text{s}^2}$$

$$P_1 - T_1 = m_1 \cdot a \Rightarrow T_1 = P_1 - m_1 \cdot a = 400 - 40 \cdot 6.16 = 154 \text{ N}$$

$$T_2 - P_3 = m_3 \cdot a \Rightarrow T_2 = P_3 + m_3 \cdot a = 100 + 10 \cdot 6.16 = 162 \text{ N}$$

68) $m = 0.15 \text{ kg}$

$v_0 = 32 \frac{\text{m}}{\text{s}}$

$v = 50 \frac{\text{m}}{\text{s}}$

$\Delta t = 0.355 \text{ s}$

a) find F ?

b) find I ?

$$\vec{I} = \vec{F} \cdot \Delta t = \Delta \vec{p} = m \cdot \Delta \vec{v} = m \cdot (\vec{v} - \vec{v}_0)$$

a) $\vec{v}_0 = 32 \cdot \vec{i}$; $\vec{v} = -50 \cdot \vec{i}$

$$\vec{F} = \frac{m \cdot (\vec{v} - \vec{v}_0)}{\Delta t} = \frac{0.15 \cdot (-50 \cdot \vec{i} - 32 \cdot \vec{i})}{0.355} =$$

$$= -34.7 \cdot \vec{i} \Rightarrow \boxed{F = 34.7 \text{ N}}$$

b) $\vec{I} = \vec{F} \cdot \Delta t = -34.7 \cdot \vec{i} \cdot 0.355 = -12.3 \cdot \vec{i} \Rightarrow \boxed{I = 12.3 \text{ N} \cdot \text{m}}$

69) $m = 2500 \text{ kg}$

$v_0 = 60 \frac{\text{km}}{\text{h}} = 16.7 \frac{\text{m}}{\text{s}}$

¿ F ?

$\Delta t = 0.5 \text{ s}$

$F = \boxed{835 \cdot 10^4 \text{ N}}$

$$\vec{I} = \vec{F} \cdot \Delta t = \Delta \vec{p} = m \cdot \Delta \vec{v} = m \cdot (\vec{v} - \vec{v}_0)$$

$$\vec{F} = \frac{m \cdot (\vec{v} - \vec{v}_0)}{\Delta t} =$$

$$= \frac{2500 \cdot (0 - 16.7 \cdot \vec{i})}{0.5} = -835 \cdot 10^4 \cdot \vec{i}$$

70) $m = 0.2 \text{ kg}$

$v_0 = 2 \frac{\text{m}}{\text{s}}$

$\Delta t = 0.1 \text{ s}$

$F = 50 \text{ N}$

¿ $\vec{v}$?

¿ $\Delta \vec{p}$?

$$\vec{I} = \vec{F} \cdot \Delta t = \Delta \vec{p} = m \cdot \Delta \vec{v} = m \cdot (\vec{v} - \vec{v}_0)$$

$$\Delta \vec{p} = \vec{F} \cdot \Delta t = 50 \cdot \vec{i} \cdot 0.1 = 5 \cdot \vec{i}$$

$\Delta p = \boxed{5 \text{ N} \cdot \text{s}}$

$$\vec{F} \cdot \Delta t = m \cdot (\vec{v} - \vec{v}_0) \Rightarrow$$

$$\Rightarrow \vec{v} - \vec{v}_0 = \frac{\vec{F} \cdot \Delta t}{m} \Rightarrow$$

$$\Rightarrow \vec{v} = \vec{v}_0 + \frac{\vec{F} \cdot \Delta t}{m} = 2 \cdot \vec{i} + \frac{50 \cdot \vec{i} \cdot 0.1}{0.2} = 27 \cdot \vec{i}$$

$v = \boxed{27 \frac{\text{m}}{\text{s}}}$

$$(71) \quad v_0 = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$$

$\vec{F}?$

$$v = 0.8 \cdot v_0 = 22.2 \frac{\text{m}}{\text{s}}$$

$$\Delta t = 0.15 \text{ s}$$

$$m = 0.057 \text{ kg}$$

$$\vec{I} = \vec{F} \cdot \Delta t = \Delta \vec{p} = m \cdot \Delta \vec{v} = m \cdot (\vec{v} - \vec{v}_0)$$

$$\vec{F} = \frac{m \cdot (\vec{v} - \vec{v}_0)}{\Delta t} =$$

$$= \frac{0.057 \cdot (-22.2 \cdot \vec{i} - 27.8 \cdot \vec{i})}{0.15} =$$

$$= -19 \cdot \vec{i} \Rightarrow \boxed{F = 19 \text{ N}}$$

$$(72) \quad m_1 = 900 \text{ kg}$$

$$v_{1i} = 120 \frac{\text{km}}{\text{h}} = 33.3 \frac{\text{m}}{\text{s}}$$

$$m_2 = 2000 \text{ kg}$$

$$v_{1f} = 30 \frac{\text{km}}{\text{h}} = 8.33 \frac{\text{m}}{\text{s}}$$

$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$900 \cdot 33.3 \cdot \vec{i} = -900 \cdot 8.33 \cdot \vec{i} + 2000 \cdot \vec{v}_{2f}$$

$$\vec{v}_{2f} = \frac{900 \cdot 33.3 \cdot \vec{i} + 900 \cdot 8.33 \cdot \vec{i}}{2000} =$$

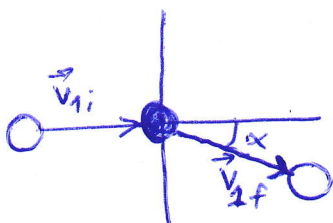
$$= 18.7 \cdot \vec{i}$$

$$v_{2f} = 18.7 \frac{\text{m}}{\text{s}} = \boxed{67.4 \frac{\text{km}}{\text{h}}}$$

$$(73) \quad v_{1i} = 2 \frac{\text{m}}{\text{s}}$$

$$v_{1f} = 0.5 \frac{\text{m}}{\text{s}}$$

$$\alpha = 30^\circ$$



$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$\vec{v}_{1f} = v_{1f} \cos \alpha \cdot \vec{i} - v_{1f} \sin \alpha \cdot \vec{j} =$$

$$= 0.5 \cdot \cos 30^\circ \cdot \vec{i} - 0.5 \cdot \sin 30^\circ \cdot \vec{j} = 0.433 \cdot \vec{i} - 0.25 \cdot \vec{j}$$

$$m \cdot 2 \cdot \vec{i} = m \cdot 0.433 \cdot \vec{i} - m \cdot 0.25 \cdot \vec{j} + m \cdot \vec{v}_{2f}$$

$$\vec{v}_{2f} = 2 \cdot \vec{i} - 0.433 \cdot \vec{i} + 0.25 \cdot \vec{j} = 1.57 \cdot \vec{i} + 0.25 \cdot \vec{j}$$

$$v_{2f} = \sqrt{1.57^2 + 0.25^2} = \boxed{1.59 \frac{\text{m}}{\text{s}}}$$

$$\cos \beta = \frac{(v_{2f})_x}{v_{2f}} = \frac{1.57}{1.59} = 0.987 \Rightarrow$$

$$(81) \Rightarrow \beta = \boxed{9.25^\circ}$$

74 $m_1 = 0'040 \text{ kg}$

$v_{1i} = 500 \frac{\text{km}}{\text{h}} = 139 \frac{\text{m}}{\text{s}}$

$m_2 = 300 \text{ kg}$

$\vec{v}_f?$

$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$

$0'040 \cdot 139 \cdot \vec{i} = (0'04 + 300) \cdot \vec{v}_f$

$\vec{v}_f = \frac{0'040 \cdot 139 \cdot \vec{i}}{300'04} = 0'0185 \cdot \vec{i} \Rightarrow$

$\Rightarrow v_f = 0'0185 \frac{\text{m}}{\text{s}} = \boxed{1'85 \frac{\text{cm}}{\text{s}}}$

75 $m_1 = 0'125 \text{ kg}$

$v_{1i} = 2 \frac{\text{m}}{\text{s}}$

$m_2 = 1 \text{ kg}$

$v_{2i} = 0$

$v_{1f} = 1'5 \frac{\text{m}}{\text{s}}$

$\vec{v}_{2f}?$

$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$

$0'125 \cdot 2 \cdot \vec{i} = -0'125 \cdot 1'5 \vec{i} + 1 \cdot \vec{v}_{2f}$

$\vec{v}_{2f} = 0'125 \cdot 2 \cdot \vec{i} + 0'125 \cdot 1'5 \cdot \vec{i} =$

$= 0'437 \cdot \vec{i}$

$v_{2f} = \boxed{0'437 \frac{\text{m}}{\text{s}}}$

76 $m_1 = 5 \text{ kg}$

$v_1 = 10 \frac{\text{m}}{\text{s}}$

$m = 50 \text{ kg}$

$m = 80 \text{ kg}$

$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$

$0 = 5 \cdot 10 \cdot \vec{i} + 130 \cdot \vec{v}_{2f}$

$\vec{v}_{2f} = \frac{-5 \cdot 10 \cdot \vec{i}}{130} = -0'385 \cdot \vec{i}$

$v_{2f} = \boxed{0'385 \frac{\text{m}}{\text{s}}}$

$$(77) \quad v = 10 \frac{\text{km}}{\text{h}} = 2.78 \frac{\text{m}}{\text{s}}$$

$$m = 0.020 \text{ kg}$$

$$v = 750 \frac{\text{m}}{\text{s}}$$

$$m = 5.8 \text{ kg}$$

$$m = 90 \text{ kg}$$

$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$95.8 \cdot 2.78 \cdot \vec{i} + 0.020 \cdot 750 \cdot \vec{i} =$$

$$= 95.8 \cdot \vec{v}_{1f} + 0.020 \cdot 750 \cdot \vec{i}$$

$$\vec{v}_{1f} = \frac{95.8 \cdot 2.78 \cdot \vec{i} + 0.02 \cdot 750 \cdot \vec{i} - 0.02 \cdot 750 \cdot \vec{i}}{95.8} = 2.6 \cdot \vec{i}$$

$$\text{Velocidad de retroceso} = 2.78 - 2.6 = \boxed{0.18 \frac{\text{m}}{\text{s}}}$$

$$(78) \quad m_1 = 800 \text{ kg}$$

$$v_1 = 40 \frac{\text{m}}{\text{s}}$$

$$m_2 = 1000 \text{ kg}$$

$$v_2 = 25 \frac{\text{m}}{\text{s}}$$

$$v_f = ?$$

$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = (m_1 + m_2) \cdot \vec{v}_f$$

$$800 \cdot 40 \cdot \vec{i} - 1000 \cdot 25 \cdot \vec{i} = 1800 \cdot \vec{v}_f$$

$$\vec{v}_f = \frac{800 \cdot 40 \cdot \vec{i} - 1000 \cdot 25 \cdot \vec{i}}{1800} =$$

$$= 3.89 \frac{\text{m}}{\text{s}} \Rightarrow v_f = \boxed{3.89 \frac{\text{m}}{\text{s}}}$$

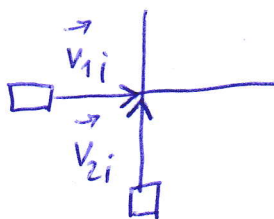
$$(79) \quad m_1 = 700 \text{ kg}$$

$$m_2 = 850 \text{ kg}$$

$$v_1 = 20 \frac{\text{m}}{\text{s}}$$

$$v_2 = 25 \frac{\text{m}}{\text{s}}$$

$$v_f = ?$$



$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = \cancel{m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}} \quad (m_1 + m_2) \cdot \vec{v}_f$$

$$700 \cdot 20 \cdot \vec{i} + 850 \cdot 25 \cdot \vec{j} =$$

$$= 1550 \cdot \vec{v}_f$$

$$\vec{v}_f = \frac{700 \cdot 20 \cdot \vec{i} + 850 \cdot 25 \cdot \vec{j}}{1550} = 9.03 \cdot \vec{i} + 13.7 \cdot \vec{j}$$

$$v_f = \sqrt{9.03^2 + 13.7^2} = 16.4 \frac{\text{m}}{\text{s}}$$

$$\cos \alpha = \frac{(v_f)_x}{v_f} = \frac{9.03}{16.4} = 0.551 \Rightarrow \alpha = \boxed{56.6^\circ}$$

80) $m = 3 \text{ ton}$

$$v = 900 \frac{\text{km}}{\text{h}} = 250 \frac{\text{m}}{\text{s}}$$

$$m = 120 \text{ kg}$$

$$v = 1500 \frac{\text{km}}{\text{h}} = 417 \frac{\text{m}}{\text{s}}$$

$$\hat{v}_{1f} ? \rightarrow \frac{\text{km}}{\text{h}}$$

$$m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$3000 \cdot 250 \cdot \vec{i} + 120 \cdot 250 \cdot \vec{i} =$$

$$= 3000 \cdot \vec{v}_{1f} + 120 \cdot 417 \cdot \vec{i}$$

$$\vec{v}_{1f} = \frac{3000 \cdot 250 \cdot \vec{i} + 120 \cdot 250 \cdot \vec{i} - 120 \cdot 417 \cdot \vec{i}}{3000} =$$

$$= 243 \cdot \vec{i}$$

$$v_{1f} = 243 \frac{\text{m}}{\text{s}} = \boxed{876 \frac{\text{km}}{\text{h}}}$$

81) $m = 2 \text{ ton}$

$$\alpha = 50^\circ$$

$$m = 120 \text{ kg}$$

$$v = 300 \frac{\text{km}}{\text{h}} = 83.3 \frac{\text{m}}{\text{s}}$$

$$a) \hat{v}_{1f} ?$$

$$b) \hat{e} ?$$

$$\mu = 0.05$$

$$c) \hat{v}_{1i} ?$$

$$a) m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$0 = (2000 - 120) \cdot \vec{v}_{1f} + 120 \cdot 83.3 \cdot \cos 50^\circ \cdot \vec{i}$$

$$\vec{v}_{1f} = -342 \cdot \vec{i} \Rightarrow v_{1f} = \boxed{342 \frac{\text{m}}{\text{s}}}$$

$$b) \sum \vec{F} = m \cdot \vec{a} \Rightarrow F_R = m \cdot a \Rightarrow$$

$$\Rightarrow \mu \cdot \phi \cdot g = \phi \cdot a \Rightarrow a = 0.05 \cdot 10 =$$

$$= 0.5 \frac{\text{m}}{\text{s}^2}$$

$$v^2 = v_0^2 - 2 \cdot a \cdot e \Rightarrow 0 = 342^2 - 2 \cdot 0.5 \cdot e \Rightarrow$$

$$\Rightarrow e = \frac{342^2}{2 \cdot 0.5} = \boxed{117 \text{ m}}$$

$$c) m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1f} + m_2 \cdot \vec{v}_{2f}$$

$$(2000 - 120) \cdot \vec{v}_{1i} + 120 \cdot \vec{v}_{2i} = (2000 - 120) \cdot 342 \cdot \vec{i} + 120 \cdot 83.3 \cdot \cos 50^\circ \cdot \vec{i}$$

$$\vec{v}_{1i} = \vec{v}_{2i} \Rightarrow \vec{v}_{1i} = \vec{v}_{2i} = \frac{321 \cdot \vec{i}}{(84)} \Rightarrow v_{1i} = \boxed{321 \frac{\text{m}}{\text{s}}}$$

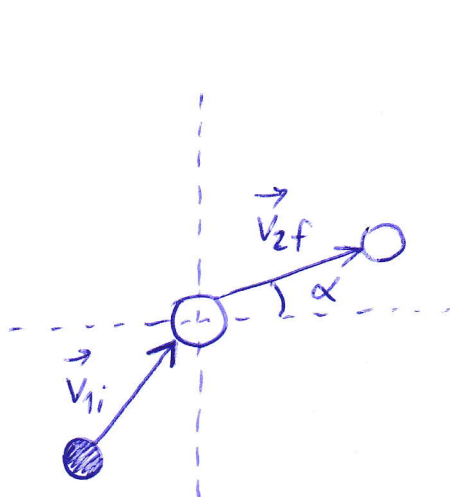
82 $\alpha = 45^\circ$

$v = 2 \frac{m}{s}$

$\alpha = 30^\circ$

$v = 1.5 \frac{m}{s}$

$\hat{v}, \beta?$



$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$m \cdot (2 \cdot \cos 45^\circ \cdot \vec{i} + 2 \cdot \sin 45^\circ \cdot \vec{j}) = m \cdot \vec{v}_{1f} + m \cdot (1.5 \cdot \cos 30^\circ \cdot \vec{i} + 1.5 \cdot \sin 30^\circ \cdot \vec{j})$$

$$\vec{v}_{1f} = 0.115 \cdot \vec{i} + 0.664 \cdot \vec{j}; v_{1f} = \sqrt{0.115^2 + 0.664^2} = 0.674 \frac{m}{s}$$

$$\cos \beta = \frac{(v_{1f})_x}{v_{1f}} = \frac{0.115}{0.674} = 0.171 \Rightarrow \beta = \boxed{80.2^\circ}$$

83 $v = 5 \frac{m}{s}$

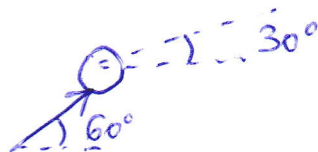
$\alpha = 60^\circ$

$m_1 = 2 \cdot m_2$

$v = 200 \frac{km}{h} = 55.6 \frac{m}{s}$

$\alpha = 30^\circ$

$\hat{v}, \alpha?$



$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_2 \vec{v}_{2f} + m_3 \vec{v}_{3f}$$

$$3 \cdot m \cdot (5 \cdot \cos 60^\circ \cdot \vec{i} + 5 \cdot \sin 60^\circ \cdot \vec{j}) =$$

$$= 2 \cdot m \cdot (55.6 \cdot \cos 30^\circ \cdot \vec{i} + 55.6 \cdot \sin 30^\circ \cdot \vec{j}) +$$

$$+ m \cdot \vec{v}_{3f}$$

$$7.5 \cdot \vec{i} + 13 \cdot \vec{j} = 96.3 \cdot \vec{i} + 55.6 \cdot \vec{j} + \vec{v}_{3f}$$

$$\vec{v}_{3f} = -88.8 \cdot \vec{i} - 42.6 \cdot \vec{j} \Rightarrow v_{3f} = 98.5 \frac{m}{s} = \boxed{355 \frac{km}{h}}$$

$$\cos \alpha = -\frac{88.8}{98.5} = -0.902 \Rightarrow \beta = \boxed{154^\circ}$$

PROBLEMAS DE TRABAJO, ENERGÍA Y POTENCIA

$$① \quad v_0 = 20 \frac{\text{km}}{\text{h}} = 5.56 \frac{\text{m}}{\text{s}}$$

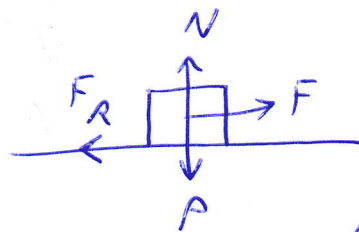
$$\mu = 0.25$$

$$v = 50 \frac{\text{km}}{\text{h}} = 13.9 \frac{\text{m}}{\text{s}}$$

$$t = 7 \text{ s}$$

$$m = 60 \text{ kg}$$

¿W?



* Primera fórmula: $W = F \cdot e \cdot \cos \alpha$
 $F_R = \mu \cdot m \cdot g = 0.25 \cdot 60 \cdot 10 = 150 \text{ N}$

$$a = \frac{\Delta v}{\Delta t} = \frac{13.9 - 5.56}{7} = 1.19 \frac{\text{m}}{\text{s}^2}$$

$$e = v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 =$$

$$= 5.56 \cdot 7 + \frac{1}{2} \cdot 1.19 \cdot 7^2 = 68.1 \text{ m}$$

$$F = F_R + m \cdot a = 150 + 60 \cdot 1.19 = 221 \text{ N}$$

$$W = F \cdot e \cdot \cos \alpha = 221 \cdot 68.1 \cdot \cos 0^\circ = \boxed{1.51 \cdot 10^4 \text{ J}}$$

* Segunda fórmula: $E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$

$$\frac{1}{2} \cdot m \cdot v_0^2 + 0 + W_F + W_F = \frac{1}{2} \cdot m \cdot v^2 + 0$$

$$\frac{1}{2} \cdot 60 \cdot 5.56^2 + 0 + W_F - 150 \cdot 68.1 = \frac{1}{2} \cdot 60 \cdot 13.9^2 + 0$$

$$927 + W_F - 1.02 \cdot 10^4 = 5800$$

$$\boxed{W_F = 1.51 \cdot 10^4 \text{ J}}$$

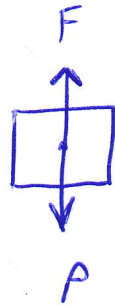
② $\hat{W}$?

$$m = 50 \text{ kg}$$

$$h = 3 \text{ m}$$

$$a) v = cte$$

$$b) a = 2 \frac{\text{m}}{\text{s}^2}$$



$$a) v = cte \Rightarrow A = 0 \Rightarrow$$

$$\Rightarrow F = P = m \cdot g = 50 \cdot 10 = 500 \text{ N}$$

$$W = F \cdot e \cdot \cos \alpha =$$

$$= 500 \cdot 3 \cdot \cos 0^\circ = \boxed{1500 \text{ J}}$$

$$b) F - P = m \cdot a \Rightarrow F = P + m \cdot a = 500 + 50 \cdot 2 = 600 \text{ N}$$

$$W = F \cdot e \cdot \cos \alpha = 600 \cdot 3 \cdot \cos 0^\circ = \boxed{1800 \text{ J}}$$

③ $F = 150 \text{ N}$

$$\alpha = 24^\circ$$

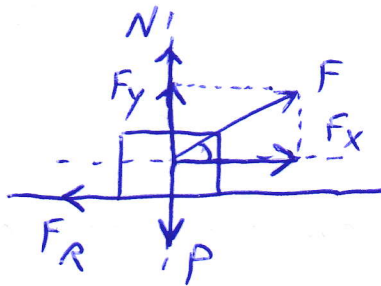
$$m = 30 \text{ kg}$$

$$t = 3 \text{ s}$$

$$v_0 = 0$$

$\hat{W}_i$?

$$\mu = 0.15$$



$$N + F_y = P$$

$$N = P - F_y =$$

$$= m \cdot g - F \cdot \sin \alpha =$$

$$= 30 \cdot 10 - 150 \cdot \sin 24^\circ = 239 \text{ N}$$

$$F_R = \mu \cdot N = 0.15 \cdot 239 = 35.9 \text{ N}$$

$$F_x - F_R = m \cdot a ; F_x = F \cdot \cos \alpha =$$

$$= 150 \cdot \cos 24^\circ = 137 \text{ N}$$

$$a = \frac{F_x - F_R}{m} = \frac{137 - 35.9}{30} = 3.37 \frac{\text{m}}{\text{s}^2}$$

$$e = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot 3.37 \cdot 3^2 = 15.2 \text{ m}$$

$$W_F = F \cdot e \cdot \cos \alpha = 150 \cdot 15.2 \cdot \cos 0^\circ = \boxed{2280 \text{ J}}$$

$$W_N = N \cdot e \cdot \cos \alpha = N \cdot e \cdot \cos 90^\circ = \boxed{0}$$

$$W_P = P \cdot e \cdot \cos \alpha = P \cdot e \cdot \cos (-90^\circ) = \boxed{0}$$

$$W_R = F_R \cdot e \cdot \cos \alpha = 35.9 \cdot 15.2 \cdot \cos 180^\circ = \boxed{-546 \text{ J}}$$

(87)

④ $\mu = 0.35$

$m = 80 \text{ kg}$

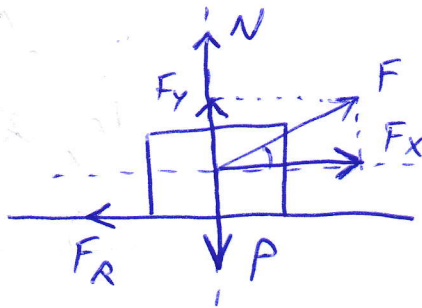
$e = 54 \text{ m}$

$t = 14 \text{ s}$

$v_0 = 0$

W?

$\alpha = 56^\circ$



$$e = \frac{1}{2} a t^2$$

$$a = \frac{2 \cdot e}{t^2} =$$

$$= \frac{2 \cdot 54}{14^2} = 0.551 \frac{\text{m}}{\text{s}^2}$$

$$N + F_y = P \Rightarrow N = P - F_y =$$

$$= m \cdot g - F \cdot \sin \alpha = 80 \cdot 10 - F \cdot \sin 56^\circ =$$

$$= 800 - 0.829 \cdot F$$

$$F_R = \mu \cdot N = 0.35 \cdot (800 - 0.829 \cdot F) = 280 - 0.29 \cdot F$$

$$F_x = F \cdot \cos \alpha = F \cdot \cos 56^\circ = 0.559 \cdot F$$

$$F_x - F_R = m \cdot a \Rightarrow 0.559 \cdot F - 280 + 0.29 \cdot F = 80 \cdot 0.551 \Rightarrow$$

$$\Rightarrow F = 382 \text{ N}$$

$$W = F \cdot e \cdot \cos \alpha = 382 \cdot 54 \cdot \cos 56^\circ = \boxed{1.95 \cdot 10^4 \text{ J}}$$

⑤ $1 \text{ kW} \cdot \text{h} = 1 \text{ kW} \cdot \text{h} \cdot \frac{1000 \cancel{\text{W}}}{1 \cancel{\text{kW}}} \cdot \frac{1 \frac{\text{J}}{\cancel{\text{s}}}}{1 \cancel{\text{W}}} \cdot \frac{3600 \cancel{\text{s}}}{1 \cancel{\text{h}}} =$

$$= 3.6 \cdot 10^6 \text{ J}$$

$$1 \text{ J} = \frac{1}{3.6 \cdot 10^6} \text{ kW} \cdot \text{h} = \boxed{2.78 \cdot 10^{-7} \text{ kW} \cdot \text{h}}$$

⑥ $m = 1300 \text{ kg}$

Pendiente = 8 %

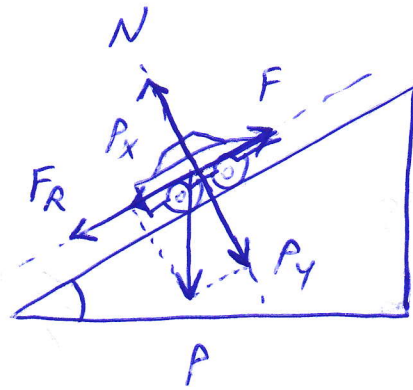
$l = 500 \text{ m}$

$v_0 = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$

$v = 70 \frac{\text{km}}{\text{h}} = 19.4 \frac{\text{m}}{\text{s}}$

¿W?

$\mu = 0.4$



$F - P_x - F_R = -m \cdot a$: por ir frenando.

$P_x = m \cdot g \cdot \sin \alpha$

pendiente = 8 % $\Rightarrow \tan \alpha = 0.08 \Rightarrow$

$\Rightarrow \alpha = 4.57^\circ$

$P_x = m \cdot g \cdot \sin \alpha = 1300 \cdot 10 \cdot \sin 4.57^\circ = 1036 \text{ N}$

$F_R = \mu \cdot m \cdot g \cdot \cos \alpha = 0.4 \cdot 1300 \cdot 10 \cdot \cos 4.57^\circ = 5183 \text{ N}$

$v^2 = v_0^2 - 2 \cdot a \cdot e \Rightarrow a = \frac{v_0^2 - v^2}{2 \cdot e} = \frac{27.8^2 - 19.4^2}{2 \cdot 500} = 0.397 \frac{\text{m}}{\text{s}^2}$

$F = P_x + F_R - m \cdot a = 1036 + 5183 - 1300 \cdot 0.397 =$
 $= 5703 \text{ N}$

$W = F \cdot e \cdot \cos \beta = 5703 \cdot 500 \cdot \cos 0^\circ = \boxed{2.85 \cdot 10^6 \text{ J}}$

$$\textcircled{7} \quad v_0 = 35 \frac{\text{km}}{\text{h}} = 9.72 \frac{\text{m}}{\text{s}}$$

$$\mu = 0.2$$

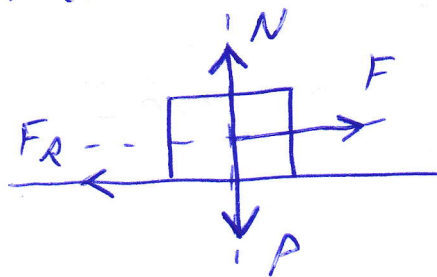
$$v = 60 \frac{\text{km}}{\text{h}} = 16.7 \frac{\text{m}}{\text{s}}$$

$$t = 8 \text{ s}$$

$$m = 90 \text{ kg}$$

$$W?$$

* Primera fórmula: $W = F \cdot e \cdot \cos \alpha$



$$F_R = \mu \cdot m \cdot g = 0.2 \cdot 900 \cdot 10 = 180 \text{ N}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{16.7 - 9.72}{8} = 0.873 \frac{\text{m}}{\text{s}^2}$$

$$e = v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 = 9.72 \cdot 8 + \frac{1}{2} \cdot 0.873 \cdot 8^2 = 106 \text{ m}$$

$$F = F_R + m \cdot a = 180 + 90 \cdot 0.873 = 259 \text{ N}$$

$$W = F \cdot e \cdot \cos \alpha = 259 \cdot 106 \cdot \cos 0^\circ = \boxed{2.75 \cdot 10^4 \text{ J}}$$

* Segunda fórmula: $E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB} \Rightarrow$

$$\Rightarrow \frac{1}{2} \cdot m \cdot v_0^2 + 0 + W_F + W_R = \frac{1}{2} \cdot m \cdot v^2 + 0 \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot 90 \cdot 9.72^2 + 0 + W_F - 180 \cdot 106 = \frac{1}{2} \cdot 90 \cdot 16.7^2 + 0 \Rightarrow$$

$$\Rightarrow 4252 + 0 + W_F - 1.91 \cdot 10^4 = 1.26 \cdot 10^4 \Rightarrow$$

$$\Rightarrow W_F = \boxed{2.75 \cdot 10^4 \text{ J}}$$

8) $v_0 = 0$

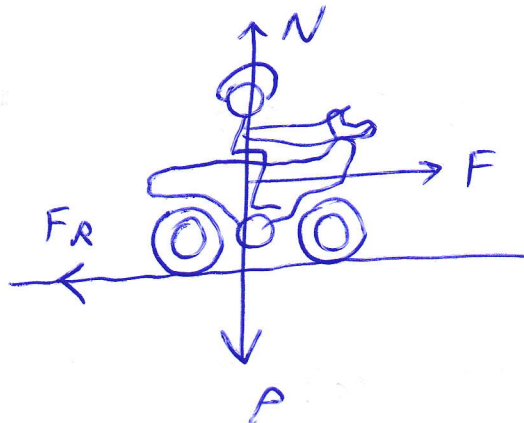
$$v = 50 \frac{\text{km}}{\text{h}} = 13.9 \frac{\text{m}}{\text{s}}$$

$$t = 3 \text{ s}$$

$$P_{\text{ot}} = ?$$

$$m = 120 \text{ kg}$$

$$\mu = 0.4$$



$$P_{\text{ot}} = \frac{F \cdot e}{t}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{13.9 - 0}{3 - 0} = 4.63 \frac{\text{m}}{\text{s}^2}$$

$$e = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot 4.63 \cdot 3^2 = 20.8 \text{ m}$$

$$F - F_R = m \cdot a; \quad F_R = \mu \cdot m \cdot g = 0.4 \cdot 120 \cdot 10 = 480 \text{ N}$$

$$F = F_R + m \cdot a = 480 + 120 \cdot 4.63 = 1036 \text{ N}$$

$$P_{\text{ot}} = \frac{F \cdot e}{t} = \frac{1036 \cdot 20.8}{3} = \boxed{7180 \text{ W}}$$

9) $m = 800 \text{ kg}$

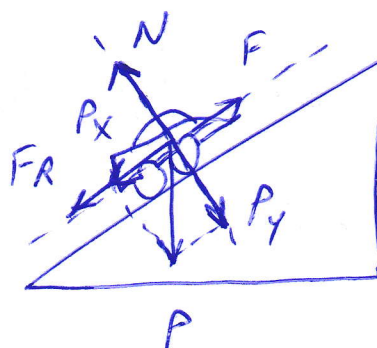
$$\alpha = 40^\circ$$

$$v = 100 \frac{\text{km}}{\text{h}} = 27.8 \frac{\text{m}}{\text{s}}$$

$$P_{\text{ot}} = ? \rightarrow \text{CV}$$

$$\mu = 0.33$$

$$1 \text{ CV} = 736 \text{ W}$$



$$P_{\text{ot}} = F \cdot v$$

$$v = cte \Rightarrow a = 0 \Rightarrow F = P_x + F_R$$

$$P_x = m \cdot g \cdot \sin \alpha = 800 \cdot 10 \cdot \sin 40^\circ = 5142 \text{ N}$$

$$F_R = \mu \cdot m \cdot g \cdot \cos \alpha = 0.33 \cdot 800 \cdot 10 \cdot \cos 40^\circ = 2022 \text{ N}$$

$$F = P_x + F_R = 5142 + 2022 = 7164 \text{ N}; \quad P_{\text{ot}} = 7164 \cdot 27.8 = 1.99 \cdot 10^5 \text{ W} = \boxed{271 \text{ CV}}$$

$$(10) Q = 20 \frac{\text{L}}{\text{s}}$$

$$h = 50 \text{ m}$$

$$P_{\text{ot}}? \rightarrow \text{CV}$$

$$1 \text{ CV} = 736 \text{ W}$$

$$P_{\text{ot}} = \frac{E}{t} = \frac{m \cdot g \cdot h}{t}$$

$$m = d \cdot V = 1 \frac{\text{kg}}{\text{L}} \cdot 20 \text{ L} = 20 \text{ kg}$$

$$P_{\text{ot}} = \frac{20 \text{ kg} \cdot 10 \cdot 50}{1} = 10^4 \text{ W} =$$

$$= 10^4 \text{ W} \cdot \frac{1 \text{ CV}}{736 \text{ W}} = \boxed{13.6 \text{ CV}}$$

$$(11) V = 6 \text{ m}^3$$

$$h = 15 \text{ m}$$

$$P_{\text{ot}} = 0.8 \text{ CV}$$

$$t?$$

$$Q?$$

$$1 \text{ CV} = 736 \text{ W}$$

$$P_{\text{ot}} = \frac{E}{t} = \frac{m \cdot g \cdot h}{t}$$

$$P_{\text{ot}} = 0.8 \text{ CV} \cdot \frac{736 \text{ W}}{1 \text{ CV}} = 589 \text{ W}$$

$$m = d \cdot V = 1000 \frac{\text{kg}}{\text{m}^3} \cdot 6 \text{ m}^3 = 6000 \text{ kg}$$

$$t = \frac{m \cdot g \cdot h}{P_{\text{ot}}} = \frac{6000 \cdot 10 \cdot 15}{589} = \boxed{1528 \text{ s}}$$

$$Q = \frac{V}{t} = \frac{6000 \text{ L}}{1528 \text{ s}} = \boxed{3.93 \frac{\text{L}}{\text{s}}}$$

12) $m = 60 \text{ kg}$

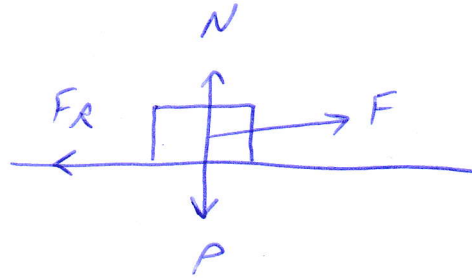
$$v_0 = 20 \frac{\text{km}}{\text{h}} = 5.56 \frac{\text{m}}{\text{s}}$$

$$\mu = 0.26$$

$$v = 80 \frac{\text{km}}{\text{h}} = 22.2 \frac{\text{m}}{\text{s}}$$

$$t = 7 \text{ s}$$

¿Pot?



$$Pot = \frac{F \cdot e}{t}$$

$$F_R = \mu \cdot m \cdot g = 0.26 \cdot 60 \cdot 10 = 156 \text{ N}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{22.2 - 5.56}{7} = 2.38 \frac{\text{m}}{\text{s}^2}$$

$$e = v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 = 5.56 \cdot 7 + \frac{1}{2} \cdot 2.38 \cdot 7^2 = 97.2 \text{ m}$$

$$F - F_R = m \cdot a \Rightarrow F = F_R + m \cdot a = 156 + 60 \cdot 2.38 = 299 \text{ N}$$

$$Pot = \frac{F \cdot e}{t} = \frac{299 \cdot 97.2}{7} = \boxed{4150 \text{ W}}$$

13) $m = 1200 \text{ kg}$

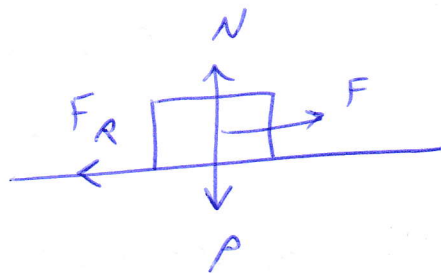
$$v_0 = 80 \frac{\text{km}}{\text{h}} = 22.2 \frac{\text{m}}{\text{s}}$$

$$v = 120 \frac{\text{km}}{\text{h}} = 33.3 \frac{\text{m}}{\text{s}}$$

$$t = 10 \text{ s}$$

¿W, Pot?

$$\mu = 0.21$$



$$Pot = \frac{W}{t}$$

$$W = F \cdot e \cdot \cos \alpha$$

$$F_R = \mu \cdot m \cdot g = 0.21 \cdot 1200 \cdot 10 = 2520 \text{ N}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{33.3 - 22.2}{10} = 1.11 \frac{\text{m}}{\text{s}^2}$$

$$F = F_R + m \cdot a = 2520 + 1200 \cdot 1.11 = 3852 \text{ N}$$

$$e = v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 = 22.2 \cdot 10 + \frac{1}{2} \cdot 1.11 \cdot 10^2 = 277 \text{ m}$$

$$W = F \cdot e \cdot \cos \alpha = 3852 \cdot 277 \cdot 1 = \boxed{1.07 \cdot 10^6 \text{ J}} \quad Pot = \frac{1.07 \cdot 10^6}{10} = \boxed{1.07 \cdot 10^5 \text{ W}}$$

(93)

(14) ¿Pot?

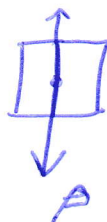
$$m = 200 \text{ kg}$$

$$h = 20 \text{ m}$$

$$t = 8 \text{ s}$$

$$P_{\text{ot}} \min \Rightarrow v = \text{cte} \Rightarrow R = 0 \Rightarrow$$

$$\Rightarrow F = P = m \cdot g_F = 200 \cdot 10 = 2000 \text{ N}$$



$$P_{\text{ot}} = \frac{F \cdot e}{t} = \frac{2000 \cdot 20}{8} = \boxed{5000 \text{ W}}$$

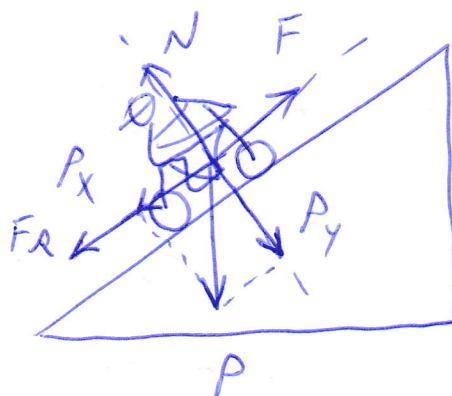
(15) ¿Pot?

$$v = 20 \frac{\text{km}}{\text{h}} = 5.56 \frac{\text{m}}{\text{s}}$$

$$\text{Pendiente} = 5\%$$

$$m_T = 90 \text{ kg}$$

$$F_R = 0.03 \cdot P$$



$$v = \text{cte} \Rightarrow R = 0 \Rightarrow F = P_x + F_R$$

$$\text{Pendiente} = 5\% \Rightarrow \tan \alpha = 0.05 \Rightarrow$$

$$\Rightarrow \alpha = 2.86^\circ$$

$$P_x = m \cdot g \cdot \sin \alpha = 90 \cdot 10 \cdot \sin 2.86^\circ = 44.9 \text{ N}$$

$$F_R = 0.03 \cdot P = 0.03 \cdot m \cdot g = 0.03 \cdot 90 \cdot 10 = 27 \text{ N}$$

$$F = P_x + F_R = 44.9 + 27 = 71.9 \text{ N}$$

$$P_{\text{ot}} = F \cdot v = 71.9 \cdot 5.56 = \boxed{400 \text{ W}}$$

16) ¿Pot? → CV

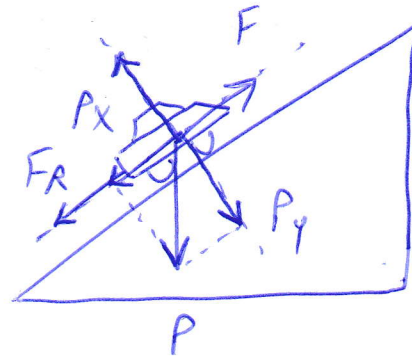
$$m = 850 \text{ kg}$$

$$\text{Pendiente} = 8\%$$

$$v = 80 \frac{\text{km}}{\text{h}} = 22\frac{1}{2} \frac{\text{m}}{\text{s}}$$

$$\mu = 0.2$$

$$1 \text{ CV} = 736 \text{ W}$$



$$Pot = F \cdot v ; F = P_x + F_R$$

$$P_x = m \cdot g \cdot \sin \alpha$$

$$\text{Pendiente} = 8\% \Rightarrow \tan \alpha = 0.08 \Rightarrow$$

$$\Rightarrow \alpha = 4.57^\circ$$

$$P_x = m \cdot g \cdot \sin \alpha = 850 \cdot 10 \cdot \sin 4.57^\circ = 677 \text{ N}$$

$$F_R = \mu \cdot m \cdot g \cdot \cos \alpha = 0.2 \cdot 850 \cdot 10 \cdot \cos 4.57^\circ = 1695 \text{ N}$$

$$F = P_x + F_R = 677 + 1695 = 2372 \text{ N}$$

$$Pot = F \cdot v = 2372 \cdot 22\frac{1}{2} = 527 \cdot 10^4 \text{ W} \cdot \frac{1 \text{ CV}}{736 \text{ W}} = \boxed{71.6 \text{ CV}}$$

17) $h = 20 \text{ m}$

¿v?

$$E_A = E_B$$

$$m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v^2$$

$$v = \sqrt{2 \cdot g \cdot h} = \sqrt{2 \cdot 10 \cdot 20} = \sqrt{400} = \boxed{20 \frac{\text{m}}{\text{s}}}$$

18 $v_0 = 300 \frac{\text{km}}{\text{h}} = 83.3 \frac{\text{m}}{\text{s}}$

¿h max.?

¿v?

$h = 50 \text{ m}$

$E_A = E_B$

$\frac{1}{2} \cdot m \cdot v^2 = m \cdot g \cdot h$

$h = \frac{v^2}{2 \cdot g} = \frac{83.3^2}{2 \cdot 10} = \boxed{347 \text{ m}}$

$E_1 = E_2 \Rightarrow \frac{1}{2} \cdot m \cdot v_1^2 = \frac{1}{2} \cdot m \cdot v_2^2 + m \cdot g \cdot h \Rightarrow$

$\Rightarrow v_1^2 = v_2^2 + 2 \cdot g \cdot h \Rightarrow v_2 = \sqrt{v_1^2 - 2 \cdot g \cdot h} =$

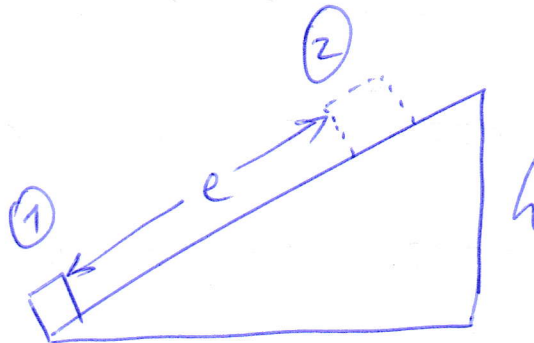
$= \sqrt{83.3^2 - 2 \cdot 10 \cdot 50} = \boxed{77.1 \frac{\text{m}}{\text{s}}}$

19 $\alpha = 45^\circ$

$v = 10 \frac{\text{m}}{\text{s}}$

$e = 4 \text{ m}$

¿μ?



$E_1 = E_2 + W_{\text{fric}} \quad E_{CA} + E_{PA} + W_{\text{FNC}} = E_{CB} + E_{PB}$

$\frac{1}{2} \cdot m \cdot v^2 = m \cdot g \cdot h + \mu \cdot m \cdot g \cdot \cos \alpha \cdot e$

$\sin \alpha = \frac{h}{e} \Rightarrow h = e \cdot \sin \alpha = 4 \cdot \sin 45^\circ = 2.83 \text{ m}$

$v^2 = 2 \cdot g \cdot h + 2 \cdot \mu \cdot g \cdot \cos \alpha \cdot e$

$\mu = \frac{v^2 - 2 \cdot g \cdot h}{2 \cdot g \cdot \cos \alpha \cdot e} = \frac{10^2 - 2 \cdot 10 \cdot 2.83}{2 \cdot 10 \cdot \cos 45^\circ \cdot 4} = \boxed{0.767}$

20 $\alpha = 60^\circ$

$m = 100 \text{ kg}$

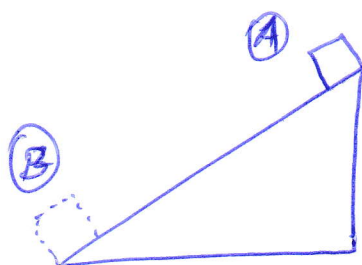
$h = 5 \text{ m}$

$\mu = 0.25$

$\vec{v}_1?$

$\vec{e}_2?$

a)



$$E_A + E_{PA} + W_{FNC} = E_B + E_{PB}$$

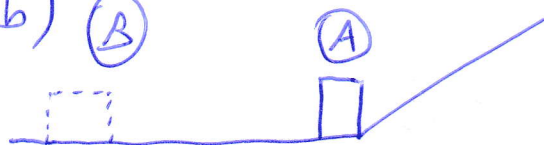
$$m \cdot g \cdot h_A - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_1 = \frac{1}{2} \cdot m \cdot v_B^2$$

$$\sin \alpha = \frac{h}{e_1} \Rightarrow e_1 = \frac{h}{\sin \alpha} = \frac{5}{\sin 60^\circ} = 5.77 \text{ m}$$

$$v_B = \sqrt{2 \cdot g \cdot (h_A - \mu \cdot \cos \alpha \cdot e_1)} =$$

$$= \sqrt{2 \cdot 10 \cdot (5 - 0.25 \cdot \cos 60^\circ \cdot 5.77)} = \boxed{9.25 \frac{\text{m}}{\text{s}}}$$

b)



$$E_A + E_{PA} + W_{FNC} = E_B + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v_A^2 - \mu \cdot m \cdot g \cdot e_2 = 0 \Rightarrow e_2 = \frac{v_A^2}{2 \cdot \mu \cdot g} =$$

$$= \frac{9.25^2}{2 \cdot 0.25 \cdot 10} = \boxed{17.1 \text{ m}}$$

(21) $m = 90 \text{ kg}$

$\alpha = 60^\circ$

$e = 8 \text{ m}$

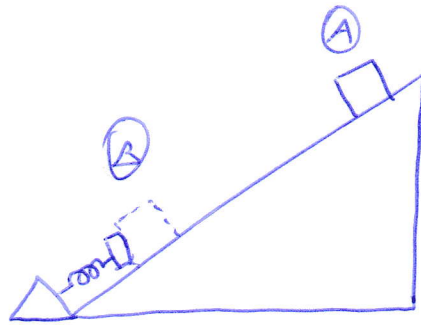
$k = 144 \cdot 10^6 \frac{\text{N}}{\text{m}}$

$\mu = 0.3$

a) $\hat{v} x?$

b) $\hat{v} v?$

a)



$$E_A^0 + E_{PA} + W_{FNC} = E_B^0 + E_{PB}$$

$$m \cdot g \cdot h - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = \frac{1}{2} \cdot k \cdot x^2$$

$$\sin \alpha = \frac{h}{e} \Rightarrow h = e \cdot \sin \alpha =$$

$$= 8 \cdot \sin 60^\circ = 6.93 \text{ m}$$

$$90 \cdot 10 \cdot 6.93 - 0.3 \cdot 90 \cdot 10 \cdot \cos 60^\circ \cdot 8 = \frac{1}{2} \cdot 144 \cdot 10^6 \cdot x^2$$

$$x = \boxed{0.0846 \text{ m}}$$

$$b) E_A^0 + E_{PA} + W_{FNC} = E_B^0 + E_{PB}$$

$$\frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot m \cdot v^2$$

$$v = \sqrt{\frac{k \cdot x^2}{m}} = x \cdot \sqrt{\frac{k}{m}} =$$

$$= 0.0846 \cdot \sqrt{\frac{144 \cdot 10^6}{90}} = \boxed{10.7 \frac{\text{m}}{\text{s}}}$$

(22) $m = 12 \text{ kg}$

$x = 0.05 \text{ m}$

$k = 3.57 \cdot 10^5 \frac{\text{N}}{\text{m}}$

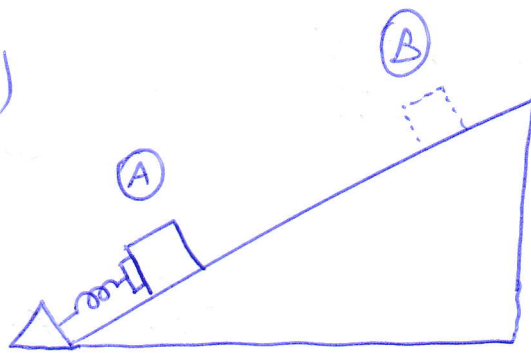
$\alpha = 40^\circ$

$\mu = 0.2$

a) $\hat{h}_{\text{max}}?$

b) $\hat{x}_0?$

a)



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

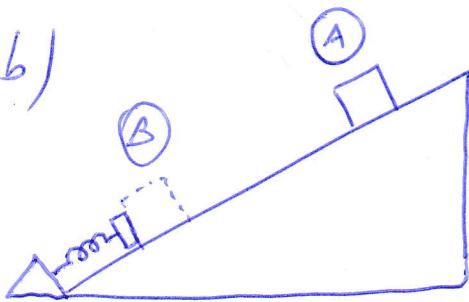
$$\frac{1}{2} \cdot k \cdot x^2 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = m \cdot g \cdot h$$

$$\sin \alpha = \frac{h}{e} \Rightarrow e = \frac{h}{\sin \alpha} = \frac{h}{\sin 40^\circ} = 1.56 \cdot h$$

$$\frac{1}{2} \cdot 3.57 \cdot 10^5 \cdot 0.05^2 - 0.2 \cdot 12 \cdot 10 \cdot \cos 40^\circ \cdot 1.56 \cdot h = 12 \cdot 10 \cdot h$$

$$446 - 28.7 \cdot h = 120 \cdot h \Rightarrow h = \boxed{3 \text{ m}}$$

b)



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$m \cdot g \cdot h_A - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = \frac{1}{2} \cdot k \cdot x^2$$

$$e = 1.56 \cdot h = 1.56 \cdot 3 = 4.68 \text{ m}$$

$$12 \cdot 10 \cdot 3 - 0.2 \cdot 12 \cdot 10 \cdot \cos 40^\circ \cdot 4.68 = \frac{1}{2} \cdot 3.57 \cdot 10^5 \cdot x^2$$

$$x = \boxed{0.0392 \text{ m}}$$

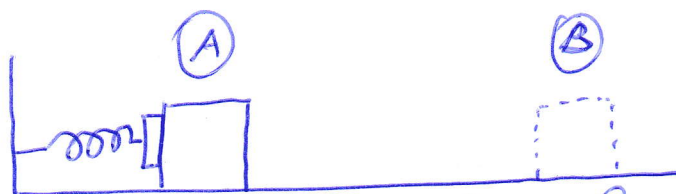
(23) $k = 1.5 \cdot 10^5 \frac{N}{m}$

$x = 0.2 \text{ m}$

$m = 25 \text{ kg}$

$\hat{e}?$

$\mu = 0.3$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot k \cdot x^2 - \mu \cdot m \cdot g \cdot e = 0$$

$$e = \frac{k \cdot x^2}{2 \cdot \mu \cdot m \cdot g} = \frac{1.5 \cdot 10^5 \cdot 0.2^2}{2 \cdot 0.3 \cdot 25 \cdot 10} = \boxed{40 \text{ m}}$$

(24) $m = 30 \text{ kg}$

$v = 15 \frac{m}{s}$

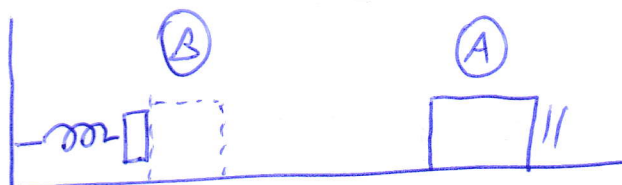
$k = 2 \cdot 10^5 \frac{N}{m}$

$e = 7 \text{ m}$

$\mu = 0.4$

$\hat{x}_0?$

$\hat{v}?$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v^2 - \mu \cdot m \cdot g \cdot e = \frac{1}{2} \cdot k \cdot x^2$$

$$\frac{1}{2} \cdot 30 \cdot 15^2 - 0.4 \cdot 30 \cdot 10 \cdot 7 = \frac{1}{2} \cdot 2 \cdot 10^5 \cdot x^2$$

$$x = \boxed{0.159 \text{ m}}$$

$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB} ; \quad \frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot m \cdot v^2 ;$$

$$v = \sqrt{\frac{k \cdot x^2}{m}} = x \cdot \sqrt{\frac{k}{m}} = 0.159 \cdot \sqrt{\frac{2 \cdot 10^5}{30}} = \boxed{13 \frac{m}{s}}$$

(100)

(25) $h = 100 \text{ m}$

b) $\dot{h}_B^?$

$$V_B = \frac{V_{\text{máx}}}{2}$$

c) $\dot{h}^?$

$$E_C = \frac{E_{C\text{máx}}}{2}$$

a) Cuando su energía potencial sea cero, a la altura del suelo.

$$b) \cancel{E_A^0} + E_{PA} = E_{CB} + \cancel{E_B^0}$$

$$M \cdot g \cdot h = \frac{1}{2} \cdot M \cdot v^2$$

$$v_{\text{máx.}} = \sqrt{2 \cdot g \cdot h} = \sqrt{2 \cdot 10 \cdot 100} =$$

$$= \sqrt{2000} = 44.7 \frac{\text{m}}{\text{s}}$$

$$V_B = \frac{V_{\text{máx.}}}{2} = \frac{44.7}{2} = 22.3 \frac{\text{m}}{\text{s}}$$

$$\cancel{E_A^0} + E_{PA} = E_{CB} + E_{PB} ; M \cdot g \cdot h_A = M \cdot g \cdot h_B + \frac{1}{2} \cdot M \cdot v_B^2 ;$$

$$2 \cdot g \cdot h_A = 2 \cdot g \cdot h_B + v_B^2$$

$$h_B = h_A - \frac{v_B^2}{2 \cdot g} = 100 - \frac{22.3^2}{2 \cdot 10} = \boxed{75.1 \text{ m}}$$

$$c) E_{C\text{máx.}} = \frac{1}{2} \cdot M \cdot v_{\text{máx.}}^2 \quad E_C = \frac{E_{C\text{máx}}}{2} \Rightarrow$$

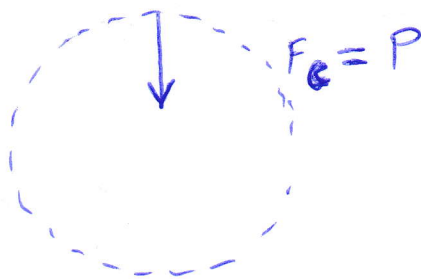
$$\Rightarrow \frac{1}{2} \cdot M \cdot v_B^2 = \frac{\frac{1}{2} \cdot M \cdot v_{\text{máx.}}^2}{2} \Rightarrow v_B = \frac{v_{\text{máx.}}}{\sqrt{2}} =$$

$$= \frac{44.7}{\sqrt{2}} = 31.6 \frac{\text{m}}{\text{s}} ; \quad \cancel{E_A^0} + E_{PA} = E_{CB} + E_{PB} ;$$

$$M \cdot g \cdot h_A = M \cdot g \cdot h_B + \frac{1}{2} \cdot M \cdot v_B^2 ; \quad h_B = h_A - \frac{v_B^2}{2 \cdot g} =$$

$$= 100 - \frac{31.6^2}{2 \cdot 10} = \boxed{50.1 \text{ m}} \quad (101)$$

(26) ¿h?
r = 10 m



$$\frac{m \cdot v^2}{r} = m \cdot g \Rightarrow v = \sqrt{r \cdot g} = \sqrt{10 \cdot 10} = 10 \frac{m}{s}$$

$$E_{cA}^0 + E_{pA} = E_{cB} + E_{pB} \Rightarrow m \cdot g \cdot h_A = m \cdot g \cdot h_B + \frac{1}{2} \cdot m \cdot v_B^2 \Rightarrow$$

$$\Rightarrow h_A = h_B + \frac{v_B^2}{2 \cdot g} = 20 + \frac{10^2}{2 \cdot 10} = \boxed{25 m}$$

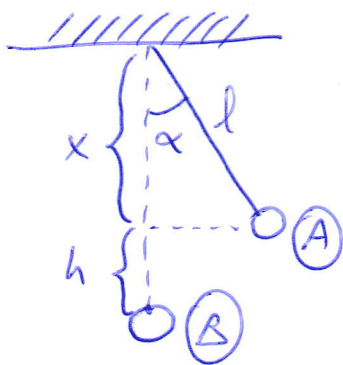
(27) m = 0.2 kg

l = 0.8 m

$\alpha = 30^\circ$

¿v?

¿T₁, T₂?



l = x + h

h = l - x

$\cos \alpha = \frac{x}{l} \Rightarrow x = l \cdot \cos \alpha$

$h = l - l \cdot \cos \alpha = l \cdot (1 - \cos \alpha)$

$$E_{cA}^0 + E_{pA} = E_{cB} + E_{pB}^0 \Rightarrow m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v^2 \Rightarrow$$

$$\Rightarrow v = \sqrt{2 \cdot g \cdot h} = \sqrt{2 \cdot g \cdot l \cdot (1 - \cos \alpha)} = \sqrt{2 \cdot 10 \cdot 0.8 \cdot (1 - \cos 30^\circ)} =$$

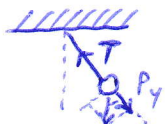
$\boxed{1.46 \frac{m}{s}}$

* En la parte más baja:

$$T - P = \frac{m \cdot v^2}{l} \Rightarrow T = P + \frac{m \cdot v^2}{l} =$$

$$= m \cdot g + \frac{m \cdot v^2}{l} = 0.2 \cdot 10 + \frac{0.2 \cdot 1.46^2}{0.8} = \boxed{2.53 N}$$

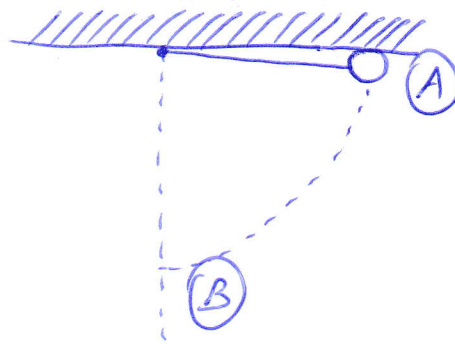
* En los extremos: $T = P_y = m \cdot g \cdot \cos \alpha =$



$$= 0.2 \cdot 10 \cdot \cos 30^\circ = \boxed{1.73 N}$$

(102)

28) $m = 0.225 \text{ kg}$
 $l = 1.4 \text{ m}$
 $\vec{v}, T?$



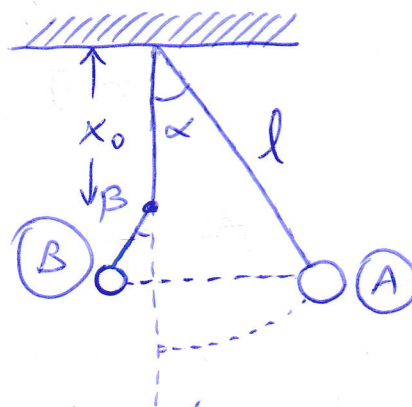
$$E_{CA} + E_{PA} = E_{CB} + E_{PB} ; m \cdot g \cdot l = \frac{1}{2} \cdot m \cdot v^2 \Rightarrow$$

$$\Rightarrow v = \sqrt{2 \cdot g \cdot l} = \sqrt{2 \cdot 10 \cdot 1.4} = \sqrt{28} = \boxed{5.29 \frac{\text{m}}{\text{s}}}$$

$$T - P = \frac{m \cdot v^2}{l} \Rightarrow T = P + \frac{m \cdot v^2}{l} = m \cdot g + \frac{m \cdot v^2}{l} =$$

$$= 0.225 \cdot 10 + \frac{0.225 \cdot 5.29^2}{1.4} = \boxed{6.75 \text{ N}}$$

29) $m = 0.25 \text{ kg}$
 $l = 0.85 \text{ m}$
 $x_0 = 0.3 \text{ m}$
 $\alpha = 60^\circ$



$$x + h = l$$

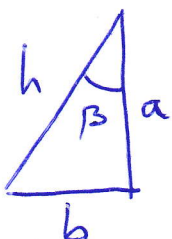
$$\cos \alpha = \frac{x}{l}$$

$$x = l \cdot \cos \alpha$$

$$h = l - x =$$

$$= l \cdot (1 - \cos \alpha)$$

$$E_{CA} + E_{PA} = E_{CB} + E_{PB} \Rightarrow h_A = h_B$$



$$a = x - x_0 = l \cdot \cos \alpha - x_0 = 0.85 \cdot \cos 60^\circ - 0.3 =$$

$$= 0.425 - 0.3 = 0.125 \text{ m}$$

$$h = l - x_0 = 0.85 - 0.3 = 0.55 \text{ m}$$

$$\cos \beta = \frac{a}{h} = \frac{0.125}{0.55} = 0.227 \Rightarrow \boxed{\beta = 76.9^\circ}$$

$$(30) \quad v_A = 500 \frac{\text{km}}{\text{h}} = 139 \frac{\text{m}}{\text{s}}$$

$$a) \quad \overset{0}{c} h_{\max}?$$

$$b) \quad \overset{0}{c} v?$$

$$h = 50 \text{ m}$$

$$c) \quad \overset{0}{c} v?$$

$$h = 50 \text{ m}$$

$$a) \quad E_{CA} + \cancel{E_{PA}^0} = \cancel{E_{CB}^0} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v^2 = m \cdot g \cdot h$$

$$h = \frac{v^2}{2 \cdot g} = \frac{139^2}{2 \cdot 10} = \boxed{966 \text{ m}}$$

$$b) \quad E_{CA} + \cancel{E_{PA}^0} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v_A^2 = \frac{1}{2} \cdot m \cdot v_B^2 + m \cdot g \cdot h_B; \quad v_A^2 = v_B^2 + 2 \cdot g \cdot h_B;$$

$$v_B = \sqrt{v_A^2 - 2 \cdot g \cdot h_B} = \sqrt{139^2 - 2 \cdot 10 \cdot 50} = \boxed{135 \frac{\text{m}}{\text{s}}}$$

$$c) \quad v = \boxed{135 \frac{\text{m}}{\text{s}}}$$

$$(31) \quad m = 5 \text{ kg}$$

$$x = 0.07 \text{ m}$$

$$k = 20000 \frac{\text{N}}{\text{m}}$$

$$\mu = 0.23$$

$$e_1 = 3 \text{ m}$$

$$\alpha = 60^\circ$$

$$\overset{0}{c} h_{\max}?$$



$$\sin \alpha = \frac{h}{e_2} \Rightarrow e_2 = \frac{h}{\sin \alpha} = \frac{h}{\sin 60^\circ} = 1.15 \cdot h$$

$$E_{CA}^0 + E_{PA} + W_{FNC} = E_{CB}^0 + E_{PB}$$

$$\frac{1}{2} \cdot k \cdot x^2 - \mu \cdot m \cdot g \cdot e_1 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_2 = m \cdot g \cdot h$$

$$\frac{1}{2} \cdot 2 \cdot 10^4 \cdot 0.07^2 - 0.23 \cdot 5 \cdot 10 \cdot 3 - 0.23 \cdot 5 \cdot 10 \cdot \cos 60^\circ \cdot 1.15 \cdot h = 5 \cdot 10 \cdot h$$

$$49 - 34.5 - 661 \cdot h = 50 \cdot h$$

$$h = \boxed{0.256 \text{ m}}$$

32) $m = 0.075 \text{ kg}$

$v = 800 \frac{\text{km}}{\text{h}} = 222 \frac{\text{m}}{\text{s}}$

$\vec{F}_R?$

$x = 0.06 \text{ m}$

$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v^2 - F_R \cdot x = 0$$

$$F_R = \frac{m \cdot v^2}{2 \cdot x} = \frac{0.075 \cdot 222^2}{2 \cdot 0.06} =$$

$$= \boxed{3.04 \cdot 10^4 \text{ N}}$$

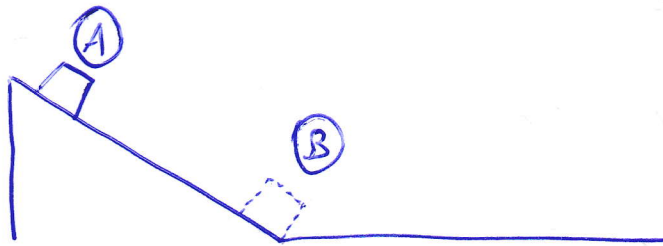
33) $\alpha = 30^\circ$

$e_1 = 12 \text{ m}$

$\mu = 0.2$

$\vec{v}?$

$e_2?$

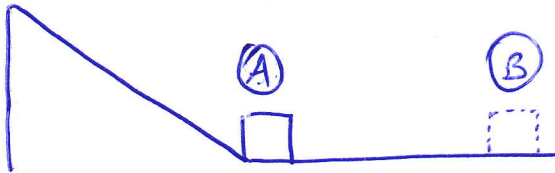


$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$m \cdot g \cdot h_A - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_1 = \frac{1}{2} \cdot m \cdot v_B^2$$

$$\sin \alpha = \frac{h}{e_1} \Rightarrow h = e_1 \cdot \sin \alpha = 12 \cdot \sin 30^\circ = 6 \text{ m}$$

$$10 \cdot 6 - 0.2 \cdot 10 \cdot \cos 30^\circ \cdot 12 = \frac{1}{2} \cdot v_B^2 \Rightarrow v_B = \boxed{8.86 \frac{\text{m}}{\text{s}}}$$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v_A^2 - \mu \cdot m \cdot g \cdot e_2 = 0$$

$$e_2 = \frac{v_A^2}{2 \cdot \mu \cdot g} = \frac{8.86^2}{2 \cdot 0.2 \cdot 10} = \boxed{19.6 \text{ m}}$$

(34) $m = 100 \text{ kg}$ | a) $\cancel{E_{CA}^0} + E_{PA} = \cancel{E_{CB}^0} + \cancel{E_{PB}^0}$
 $h = 8 \text{ m}$
 $x = 0.5 \text{ m}$
 a) $\hat{E}_{CB}?$
 b) $\hat{F}_R?$

$$E_{CB} = E_{PA} = m \cdot g \cdot h_A =$$

$$= 100 \cdot 10 \cdot 8 = \boxed{8000 \text{ J}}$$

b) $\cancel{E_{CA}^0} + E_{PA} + W_{FNC} = \cancel{E_{CB}^0} + \cancel{E_{PB}^0}$

$$m \cdot g \cdot h_A - F_R \cdot x = 0 \Rightarrow F_R = \frac{m \cdot g \cdot h_A}{x} =$$

$$= \frac{100 \cdot 10 \cdot 8}{0.5} = \boxed{1.6 \cdot 10^4 \text{ N}}$$

(35) $m = 2 \text{ kg}$ | $\cancel{E_{CA}^0} + E_{PA} + W_{FNC} = \cancel{E_{CB}^0} + \cancel{E_{PB}^0}$
 $h = 10 \text{ m}$
 $\hat{F}_R?$
 $x = 0.2 \text{ m}$

$$m \cdot g \cdot h - F_R \cdot x = 0$$

$$F_R = \frac{m \cdot g \cdot h}{x} = \frac{2 \cdot 10 \cdot 10}{0.2} =$$

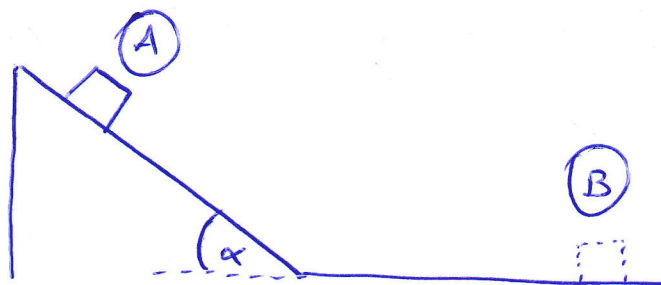
$$= \frac{200}{0.2} = \boxed{1000 \text{ N}}$$

36 $h = 10 \text{ m}$

$\alpha = 40^\circ$

$\mu = 0.3$

$\mu = 0.3$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$0 + \mu \cdot g \cdot h_A - \mu \cdot \mu \cdot g \cdot \cos \alpha \cdot e_1 - \mu \cdot \mu \cdot g \cdot e_2 = 0$$

$$\sin \alpha = \frac{h_A}{e_1} \Rightarrow e_1 = \frac{h_A}{\sin \alpha} = \frac{10}{\sin 40^\circ} = 15.6 \text{ m}$$

$$10 \cdot 10 - 0.3 \cdot 10 \cdot \cos 40^\circ \cdot 15.6 - 0.3 \cdot 10 \cdot e_2 = 0$$

$$e_2 = \boxed{21.4 \text{ m}}$$

37 $a = 3 \text{ m}$

$k_1 = 20000 \frac{\text{N}}{\text{m}}$

$k_2 = 25000 \frac{\text{N}}{\text{m}}$

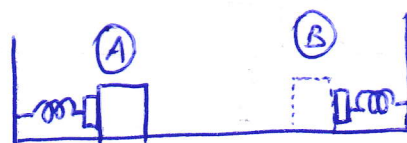
$m = 5 \text{ kg}$

$x_1 = 0.14 \text{ m}$

$\mu = 0.25$

a) $\mu = 0.25$

b) $\mu = 0.25$



$$a) E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot k_1 \cdot x_1^2 - \mu \cdot m \cdot g \cdot a = \frac{1}{2} \cdot m \cdot v^2$$

$$\frac{1}{2} \cdot 20000 \cdot 0.14^2 - 0.25 \cdot 5 \cdot 10 \cdot 3 = \frac{1}{2} \cdot 5 \cdot v^2$$

$$v = \boxed{7.96 \frac{\text{m}}{\text{s}}}$$

$$b) E_{CA} + E_{PA} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v_A^2 = \frac{1}{2} \cdot k_2 \cdot x^2 ; x = v_A \cdot \sqrt{\frac{m}{k_2}} =$$

$$= 7.96 \cdot \sqrt{\frac{5}{25000}} = \boxed{0.113 \text{ m}}$$

38) $m = 0.020 \text{ kg}$ | $E_A + E_{PA} = E_B + E_{PB}$

$k = 100 \frac{\text{N}}{\text{m}}$ | $\frac{1}{2} \cdot k \cdot x^2 = m \cdot g \cdot h$

$x = 0.05 \text{ m}$ | $h = \frac{k \cdot x^2}{2 \cdot m \cdot g} = \frac{100 \cdot 0.05^2}{2 \cdot 0.02 \cdot 10} =$

$\vec{v}, h?$ | $= \boxed{0.625 \text{ m}}$

$E_A + E_{PA} = E_B + E_{PB} \Rightarrow \frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot m \cdot v^2 \Rightarrow$

$\Rightarrow v = x \cdot \sqrt{\frac{k}{m}} = 0.05 \cdot \sqrt{\frac{100}{0.02}} = \boxed{3.54 \frac{\text{m}}{\text{s}}}$

39) $m = 2 \text{ kg}$

$v = 5 \frac{\text{m}}{\text{s}}$

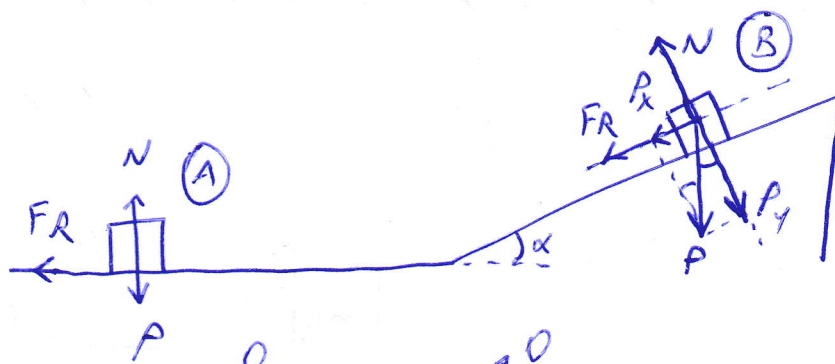
$e_1 = 0.2 \text{ m}$

$\alpha = 30^\circ$

$\mu = 0.1$

$\vec{h}?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$E_{CA} + E_{PA} + W_{FNC} = E_B + E_{PB}$

$\frac{1}{2} \cdot m \cdot v_A^2 - \mu \cdot m \cdot g \cdot e_1 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_2 = m \cdot g \cdot h_B$

$\sin \alpha = \frac{h}{e_2} \Rightarrow e_2 = \frac{h}{\sin \alpha} = \frac{h}{\sin 30^\circ} = 2 \cdot h$

$\frac{1}{2} \cdot 5^2 - 0.1 \cdot 9.8 \cdot 0.2 - 0.1 \cdot 9.8 \cdot \cos 30^\circ \cdot 2 \cdot h = 9.8 \cdot h$

$12.5 - 0.196 - 1.7 \cdot h = 9.8 \cdot h \Rightarrow h = \boxed{1.07 \text{ m}}$

40 $m = 20 \text{ kg}$

$v_0 = 0$

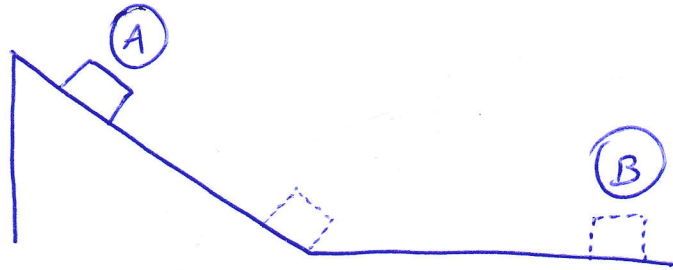
$\alpha = 30^\circ$

$e_1 = 15 \text{ m}$

$e_2 = 20 \text{ m}$

μ ?

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$E_A^0 + E_{PA} + W_{FNC} = E_B^0 + E_{PB}^0$$

$$\sin \alpha = \frac{h}{e_1} \Rightarrow h = e_1 \cdot \sin \alpha =$$

$$= 15 \cdot \sin 30^\circ = 7.5 \text{ m}$$

$$\mu \cdot g \cdot h_A - \mu \cdot g \cdot e_2 = 0$$

$$\mu = \frac{h_A}{e_2} = \frac{7.5}{20} = \boxed{0.375}$$

41 $m = 10 \text{ kg}$

$v_0 = 0$

$F = 40 \text{ N}$

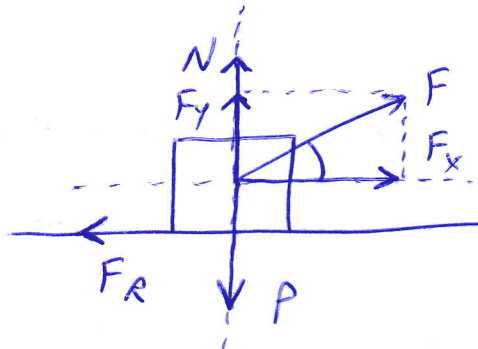
$\alpha = 60^\circ$

$\mu = 0.2$

ΔE_c ?

$e = 0.5 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$N + F_y = P \Rightarrow N = P - F_y =$$

$$= m \cdot g - F \cdot \sin \alpha = 10 \cdot 9.8 - 40 \cdot \sin 60^\circ = 63.4 \text{ N}$$

$$F_R = \mu \cdot N = 0.2 \cdot 63.4 = 12.7 \text{ N}$$

$$F_x - F_R = m \cdot a ; F_x = F \cdot \cos \alpha =$$

$$= 40 \cdot \cos 60^\circ = 20 \text{ N}$$

$$a = \frac{F_x - F_R}{m} = \frac{20 - 12.7}{10} = 0.73 \frac{\text{m}}{\text{s}^2} ; v^2 = v_0^2 + 2 \cdot a \cdot e =$$

$$= 0 + 2 \cdot 0.73 \cdot 0.5 = 0.73 \Rightarrow \Delta E_c = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot 10 \cdot 0.73 = \boxed{3.65 \text{ J}}$$

(109)

42 $m = 1 \text{ kg}$

$\alpha = 30^\circ$

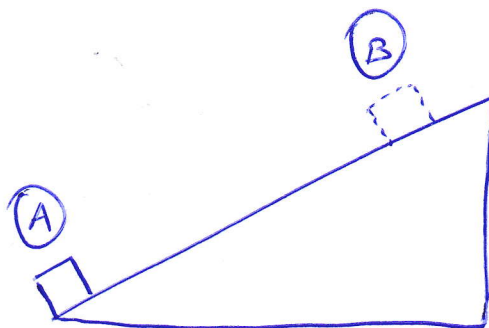
$v = 10 \frac{\text{m}}{\text{s}}$

$\mu = 0.3$

$\hat{r} h?$

$\hat{r} W_R?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v^2 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = m \cdot g \cdot h$$

$$\sin \alpha = \frac{h}{e} \Rightarrow e = \frac{h}{\sin \alpha} = \frac{h}{\sin 30^\circ} = 2 \cdot h$$

$$\frac{1}{2} \cdot 10^2 - 0.3 \cdot 9.8 \cdot \cos 30^\circ \cdot 2 \cdot h = 9.8 \cdot h$$

$$50 - 5.09 \cdot h = 9.8 \cdot h \Rightarrow h = \boxed{3.36 \text{ m}}$$

$$W_R = -\mu \cdot m \cdot g \cdot \cos \alpha \cdot e = -0.3 \cdot 1 \cdot 9.8 \cdot \cos 30^\circ \cdot 2 \cdot 3.36 = \boxed{-17.1 \text{ J}}$$

43 $m = 2 \text{ kg}$

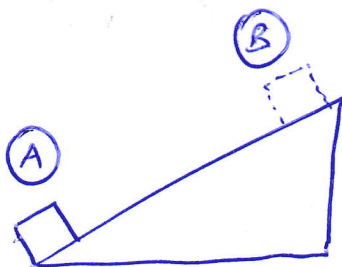
$\mu = 0.3$

$\alpha = 30^\circ$

$v = 6 \frac{\text{m}}{\text{s}}$

$\hat{r} h?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$E_{CA} + E_{PA} + W_{FNC} = E_{CB} + E_{PB}$$

$$\frac{1}{2} \cdot m \cdot v_A^2 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = m \cdot g \cdot h$$

$$\sin \alpha = \frac{h}{e} \Rightarrow e = \frac{h}{\sin \alpha} = \frac{h}{\sin 30^\circ} = 2 \cdot h$$

$$\frac{1}{2} \cdot 6^2 - 0.3 \cdot 9.8 \cdot \cos 30^\circ \cdot 2 \cdot h = 9.8 \cdot h$$

$$18 - 5.09 \cdot h = 9.8 \cdot h \Rightarrow h = \boxed{1.21 \text{ m}}$$

44) $m = 3 \text{ kg}$

$v = 20 \frac{\text{m}}{\text{s}}$

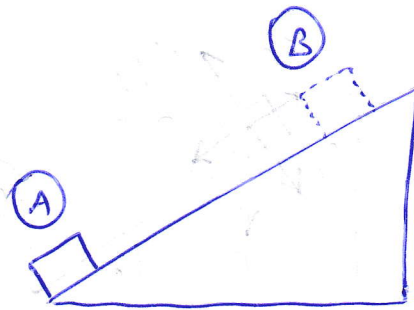
$\alpha = 60^\circ$

$\mu = 0.3$

¿e?

¿W_R?

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$\sin \alpha = \frac{h}{e}$

$h = e \cdot \sin \alpha$

$E_{CA} + E_{PA}^0 + W_{FNC} = E_{CB}^0 + E_{PB}$

$\frac{1}{2} \cdot m \cdot v^2 - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e = m \cdot g \cdot e \cdot \sin \alpha$

$\frac{1}{2} \cdot 20^2 - 0.3 \cdot 9.8 \cdot \cos 60^\circ \cdot e = 9.8 \cdot e \cdot \sin 60^\circ$

$200 - 1.47 \cdot e = 8.49 \cdot e \Rightarrow e = \boxed{20.1 \text{ m}}$

$W_R = -\mu \cdot m \cdot g \cdot \cos \alpha \cdot e = -0.3 \cdot 3 \cdot 9.8 \cdot \cos 60^\circ \cdot 20.1 = \boxed{-88.6 \text{ J}}$

45) $m = 5 \text{ kg}$

$F = 30 \text{ N}$

$\alpha = 60^\circ$

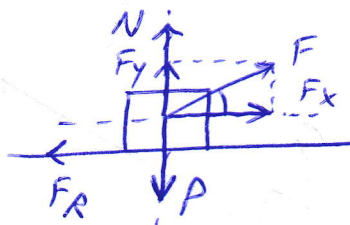
$\mu = 0.2$

¿Fuerzas?

¿ΔEc?

$e = 0.5 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$P = m \cdot g = 5 \cdot 9.8 = \boxed{49 \text{ N}}$

$F_x = F \cdot \cos \alpha = 30 \cdot \cos 60^\circ = \boxed{15 \text{ N}}$

$F_y = F \cdot \sin \alpha = 30 \cdot \sin 60^\circ = \boxed{26 \text{ N}}$

$N = P - F_y = 49 - 26 = \boxed{23 \text{ N}}$

$F_R = \mu \cdot N = 0.2 \cdot 23 = \boxed{4.6 \text{ N}}$

$F_x - F_R = m \cdot a \Rightarrow a = \frac{F_x - F_R}{m} = \frac{15 - 4.6}{5} = \boxed{2.08 \frac{\text{m}}{\text{s}^2}}$

$v^2 = v_0^2 + 2 \cdot a \cdot e = 0 + 2 \cdot 2.08 \cdot 0.5 = \boxed{2.08}$

$\Delta E_c = \frac{1}{2} \cdot m \cdot (v^2 - v_0^2) = \frac{1}{2} \cdot 5 \cdot (2.08 - 0) = \boxed{5.2 \text{ J}}$

46 $v_0 = 0$

$\alpha = 30^\circ$

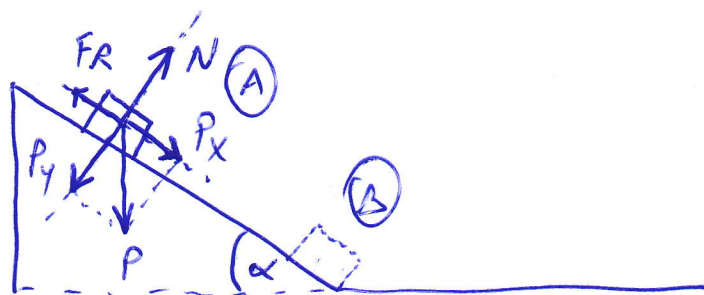
$e_1 = 2 \text{ m}$

$v = 4 \frac{\text{m}}{\text{s}}$

$e_2 = 4 \text{ m}$

$\mu_1?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$E_{C_A} + E_{P_A} + W_{FNC} = E_{C_B} + E_{P_B}$$

$$\mu \cdot g \cdot h - \mu_1 \cdot \mu \cdot g \cdot \cos \alpha \cdot e_1 = \frac{1}{2} \mu \cdot v^2$$

$$\sin \alpha = \frac{h}{e_1} \Rightarrow h = e_1 \cdot \sin \alpha = 2 \cdot \sin 30^\circ = 1 \text{ m}$$

$$9.8 \cdot 1 - \mu_1 \cdot 9.8 \cdot \cos 30^\circ \cdot 2 = \frac{1}{2} \cdot 4^2$$

$$9.8 - 17 \cdot \mu_1 = 8 \Rightarrow \mu_1 = \frac{9.8 - 8}{17} = \boxed{0.106}$$

47 $m = 2 \text{ kg}$

$\alpha = 30^\circ$

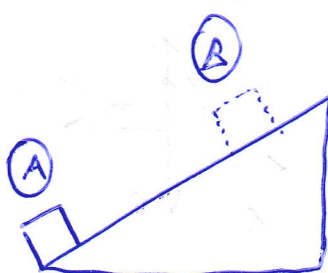
$v = 10 \frac{\text{m}}{\text{s}}$

$e = 8 \text{ m}$

$\mu?$

$\Delta E_C, \Delta E_P, \Delta E_M?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$E_{C_A} + E_{P_A} + W_{FNC} = E_{C_B} + E_{P_B}$$

$$\sin \alpha = \frac{h}{e}$$

$$h = e \cdot \sin \alpha = 8 \cdot \sin 30^\circ = 4 \text{ m}$$

$$\frac{1}{2} \mu \cdot v^2 - \mu \cdot \mu \cdot g \cdot \cos \alpha \cdot e = \mu \cdot g \cdot h$$

$$\frac{1}{2} \cdot 10^2 - \mu \cdot 9.8 \cdot \cos 30^\circ \cdot 8 = 9.8 \cdot 4$$

$$\mu = \boxed{0.159}$$

$$\Delta E_C = \frac{1}{2} m \cdot (v^2 - v_0^2) = \frac{1}{2} \cdot 2 \cdot (0 - 10^2) = \boxed{-100 \text{ J}}$$

$$\Delta E_P = m \cdot g \cdot \Delta h = 2 \cdot 9.8 \cdot 4 = \boxed{78.4 \text{ J}}$$

$$\Delta E_M = \Delta E_C + \Delta E_P = -100 + 78.4 = \boxed{-21.6 \text{ J}}$$

48 $\alpha = 30^\circ$

$m = 100 \text{ kg}$

$v = cte$

$\mu = 0.2$

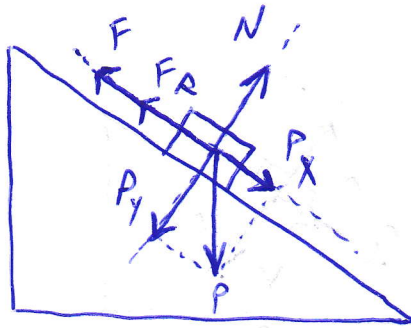
$\hat{?} F?$

$\hat{?} \Delta E_p?$

$e = 20 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$

$\Delta E_p = m \cdot g \cdot \Delta h = 100 \cdot 9.8 \cdot (0 - 10) = \boxed{-9800 \text{ J}}$



$v = cte \Rightarrow R = 0 \Rightarrow$

$\Rightarrow F + F_R = P_x$

$$F = P_x - F_R = m \cdot g \cdot \sin \alpha - \mu \cdot m \cdot g \cdot \cos \alpha =$$

$$= 100 \cdot 9.8 \cdot \sin 30^\circ - 0.2 \cdot 100 \cdot 9.8 \cdot \cos 30^\circ =$$

$$= 490 - 170 = \boxed{320 \text{ N}}$$

$\sin \alpha = \frac{h}{e} \Rightarrow h = e \cdot \sin \alpha = 20 \cdot \sin 30^\circ = 10 \text{ m}$

49 $m = 5 \text{ kg}$

$v = cte$

$F = 20 \text{ N}$

$\alpha = 60^\circ$

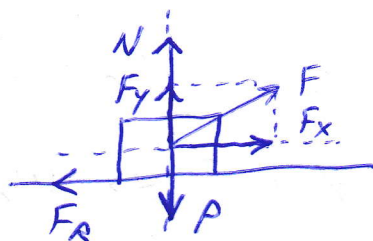
$\hat{?} \text{ Fuerzas?}$

$\hat{?} \mu?$

$\hat{?} W_T?$

$e = 2 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$P = m \cdot g = 5 \cdot 9.8 = \boxed{49 \text{ N}}$

$F_x = F \cdot \cos \alpha =$
 $= 20 \cdot \cos 60^\circ = \boxed{10 \text{ N}}$

$F_y = F \cdot \sin \alpha = 20 \cdot \sin 60^\circ = \boxed{17.3 \text{ N}}$

$N = P - F_y = 49 - 17.3 = \boxed{31.7 \text{ N}}$

$v = cte \Rightarrow R = 0 \Rightarrow F_x = F_R \Rightarrow F_x = \mu \cdot N \Rightarrow$

$\Rightarrow \mu = \frac{F_x}{N} = \frac{10}{31.7} = \boxed{0.316}$

$F_R = \mu \cdot N = 0.316 \cdot 31.7 = \boxed{10 \text{ N}}$

$W_T = W_F + W_R = F \cdot e \cdot \cos \alpha + F_R \cdot e \cdot \cos \alpha =$

$= 20 \cdot 2 \cdot \cos 0^\circ + 10 \cdot 2 \cdot \cos 180^\circ = 40 - 20 = \boxed{20 \text{ J}}$

(50) $m = 2 \text{ kg}$

$\mu = 0.2$

$\alpha = 30^\circ$

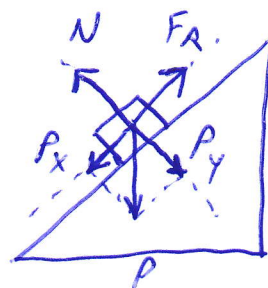
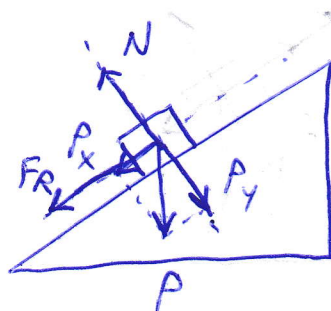
$v = 6 \frac{\text{m}}{\text{s}}$

$v = 4.2 \frac{\text{m}}{\text{s}}$

¿Fuerzas?

¿ W_R ?

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$P = m \cdot g = 2 \cdot 9.8 = \boxed{19.6 \text{ N}}$

$P_x = m \cdot g \cdot \sin \alpha = 2 \cdot 9.8 \cdot \sin 30^\circ = \boxed{9.8 \text{ N}}$

$P_y = m \cdot g \cdot \cos \alpha = 2 \cdot 9.8 \cdot \cos 30^\circ = \boxed{17 \text{ N}}$

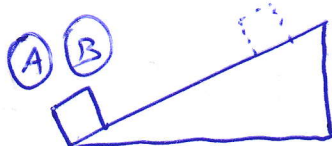
$N = P_y = \boxed{17 \text{ N}}$

$F_R = \mu \cdot N = 0.2 \cdot 17 = \boxed{3.4 \text{ N}}$

$E_{CA} + E_{PA}^0 + W_{FNC} = E_{CB} + E_{PB}^0$

$\frac{1}{2} \cdot m \cdot v_A^2 + 2 \cdot W_R = \frac{1}{2} \cdot m \cdot v_B^2$

$\frac{1}{2} \cdot 2 \cdot 6^2 + 2 \cdot W_R = \frac{1}{2} \cdot 2 \cdot 4.2^2 \Rightarrow W_R = \frac{-36 + 4.2^2}{2} = \boxed{-9.18 \text{ J}}$



(51) $m = 5 \text{ kg}$

$v_0 = 0$

$\alpha = 30^\circ$

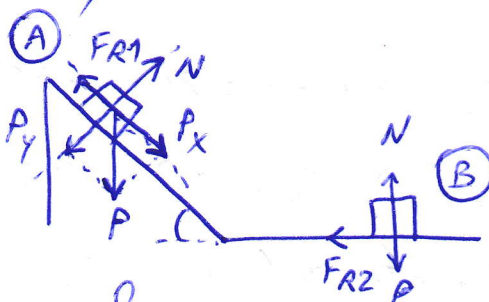
$h = 0.4 \text{ m}$

$\mu = 0.3$

¿ e_2 ?

¿ W_T ?

$g = 10 \frac{\text{m}}{\text{s}^2}$



$\sin \alpha = \frac{h}{e_1} \Rightarrow$

$\Rightarrow e_1 = \frac{h}{\sin \alpha} = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ m}$

$E_{CA}^0 + E_{PA} + W_{FNC} = E_{CB}^0 + E_{PB}^0$

$m \cdot g \cdot h - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_1 - \mu \cdot m \cdot g \cdot e_2 = 0$

$e_2 = \frac{h - \mu \cdot \cos \alpha \cdot e_1}{\mu} = \frac{0.4 - 0.3 \cdot \cos 30^\circ \cdot 0.8}{0.3} =$

$= \boxed{0.641 \text{ m}}$

$E_{CA}^0 + E_{PA} + W_{FNC} = E_{CB}^0 + E_{PB}^0$; $m \cdot g \cdot h - \mu \cdot m \cdot g \cdot \cos \alpha \cdot e_1 = E_{CB}$

$E_{CB} = 5 \cdot 10 \cdot 0.4 - 0.3 \cdot 5 \cdot 10 \cdot \cos 30^\circ \cdot 0.8 = 9.61 \text{ J}$; $W_T = \Delta E_C = \boxed{9.61 \text{ J}}$



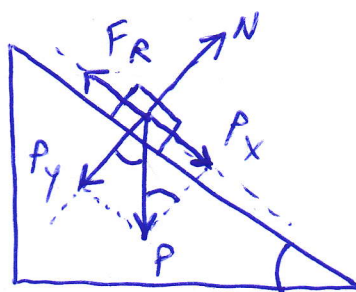
52) $m = 2 \text{ kg}$

$h = 5 \text{ m}$

$v = 6 \frac{\text{m}}{\text{s}}$

$\vec{W}_P, \vec{W}_R?$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$\cancel{E_{CA}} + E_{PA} + W_{FNC} = E_{CB} + \cancel{E_{PB}}$$

$$m \cdot g \cdot h + W_R = \frac{1}{2} \cdot m \cdot v^2$$

$$W_R = \frac{1}{2} \cdot m \cdot v^2 - m \cdot g \cdot h = \frac{1}{2} \cdot 2 \cdot 6^2 - 2 \cdot 9.8 \cdot 5 = \boxed{-62 \text{ J}}$$

$$W_T = \Delta E_C = \frac{1}{2} \cdot m \cdot (v^2 - v_0^2) = \frac{1}{2} \cdot 2 \cdot (6^2 - 0^2) = 36 \text{ J}$$

$$W_T = W_{FC} + W_{FNC} \Rightarrow W_{FC} = W_T - W_{FNC} = 36 - (-62) = \boxed{98 \text{ J}}$$

53) $m = 200 \text{ kg}$

$v = \text{cte}$

$\alpha = 30^\circ$

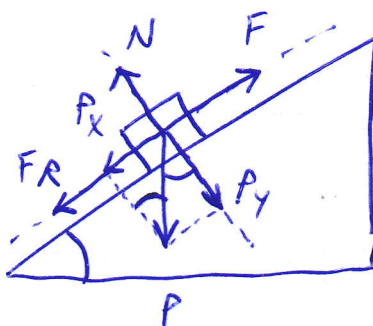
$\mu = 0.1$

$\vec{F}?$

$\Delta E_P?$

$e = 20 \text{ m}$

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$v = \text{cte} \Rightarrow a = 0 \Rightarrow$

$\Rightarrow F = P_x + F_R$

$$F = m \cdot g \cdot \sin \alpha + \mu \cdot m \cdot g \cdot \cos \alpha =$$

$$= 200 \cdot 9.8 \cdot \sin 30^\circ + 0.1 \cdot 200 \cdot 9.8 \cdot \cos 30^\circ =$$

$$= 980 + 170 = \boxed{1150 \text{ N}}$$

$$\sin \alpha = \frac{h}{e}$$

$$h = e \cdot \sin \alpha = 20 \cdot \sin 30^\circ = 10 \text{ m}$$

$$\Delta E_P = m \cdot g \cdot \Delta h = 200 \cdot 9.8 \cdot 10 = \boxed{1.96 \cdot 10^4 \text{ J}}$$

PROBLEMAS DE MOVIMIENTO ARMÓNICO SIMPLE

① $T = \sqrt{3} \text{ s}$

$\varphi_0 = 30^\circ = \frac{\pi}{6} \text{ rad}$

$\dot{y}, v, a?$

$t = 10 \text{ s}$

$A = \sqrt{2} \text{ m}$

$\dot{y}?$

$$y = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$$\omega = \frac{2 \cdot \pi}{T} = \frac{2 \cdot \pi}{\sqrt{3}} = \frac{20 \cdot \pi}{3} \frac{\text{rad}}{\text{s}}$$

$$y = \sqrt{2} \cdot \sin\left(\frac{20 \cdot \pi}{3} \cdot t + \frac{\pi}{6}\right)$$

$$v = \frac{dy}{dt} = \sqrt{2} \cdot \frac{20 \cdot \pi}{3} \cdot \cos\left(\frac{20 \cdot \pi}{3} \cdot t + \frac{\pi}{6}\right) =$$

$$= 419 \cdot \cos\left(\frac{20 \cdot \pi \cdot 10}{3} + \frac{\pi}{6}\right) = \boxed{-363 \frac{\text{m}}{\text{s}}}$$

$$a = \frac{dv}{dt} = -419 \cdot \frac{20 \cdot \pi}{3} \cdot \sin\left(\frac{20 \cdot \pi \cdot t}{3} + \frac{\pi}{6}\right) =$$

$$= -878 \cdot \sin\left(\frac{20 \cdot \pi \cdot 10}{3} + \frac{\pi}{6}\right) = \boxed{-439 \frac{\text{m}}{\text{s}^2}}$$

$$v = 0 \Rightarrow \cos\left(\frac{20 \cdot \pi \cdot t}{3} + \frac{\pi}{6}\right) = 0 \Rightarrow \frac{20 \cdot \pi \cdot t}{3} + \frac{\pi}{6} = (2n+1) \cdot \frac{\pi}{2};$$

$$40 \cdot t + 1 = 6n + 3 \Rightarrow 40 \cdot t = 6n + 2; t = \frac{6n+2}{40} = \boxed{\frac{3n+1}{20}}$$

$$a = 0 \Rightarrow \sin\left(\frac{20 \cdot \pi \cdot t}{3} + \frac{\pi}{6}\right) = 0 \Rightarrow \frac{20 \cdot \pi \cdot t}{3} + \frac{\pi}{6} = n \cdot \pi;$$

$$40 \cdot t + 1 = 6 \cdot n \Rightarrow \boxed{t = \frac{6n-1}{40}}$$

$$(2) f = 40 \text{ Hz}$$

$$\varphi_0 = 45^\circ = \frac{\pi}{4} \text{ rad}$$

$$A = 0.3 \text{ m}$$

$$\dot{x}, v, a? \quad t = 20 \text{ s}$$

$$\dot{x}?$$

$$x = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$$\omega = 2\pi \cdot f = 2\pi \cdot 40 = 80\pi \frac{\text{rad}}{\text{s}}$$

$$x = 0.3 \cdot \sin\left(80\pi \cdot t + \frac{\pi}{4}\right)$$

$$x(t=20 \text{ s}) = 0.3 \cdot \sin\left(80\pi \cdot 20 + \frac{\pi}{4}\right) =$$

$$= \boxed{0.212 \text{ m}}$$

$$v = \frac{dx}{dt} = 0.3 \cdot 80\pi \cdot \cos\left(80\pi \cdot t + \frac{\pi}{4}\right) =$$

$$= 75.4 \cdot \cos\left(80\pi \cdot t + \frac{\pi}{4}\right)$$

$$v(t=20 \text{ s}) = 75.4 \cdot \cos\left(80\pi \cdot 20 + \frac{\pi}{4}\right) = \boxed{53.3 \frac{\text{m}}{\text{s}}}$$

$$a = \frac{dv}{dt} = -75.4 \cdot 80\pi \cdot \sin\left(80\pi \cdot t + \frac{\pi}{4}\right) =$$

$$= -1.9 \cdot 10^4 \cdot \sin\left(80\pi \cdot 20 + \frac{\pi}{4}\right) = \boxed{-1.34 \cdot 10^4 \frac{\text{m}}{\text{s}^2}}$$

$$a = 0 \Rightarrow \sin\left(80\pi \cdot t + \frac{\pi}{4}\right) = 0 \Rightarrow 80\pi \cdot t + \frac{\pi}{4} = n \cdot \pi ;$$

$$320 \cdot t + 1 = 4 \cdot n \Rightarrow \boxed{t = \frac{4n-1}{320}}$$

3) a) $m = 10 \text{ kg}$

$$y = A \cdot \sin(\omega \cdot t + \varphi_0) ; A = y_{\max} = 0.04 \text{ m}$$

$$T = 12 \text{ s} ; \omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$y_0 = A \cdot \sin \varphi_0 \Rightarrow 0.02 = 0.04 \cdot \sin \varphi_0 \Rightarrow \sin \varphi_0 = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{6} \text{ rad}$$

$$y = 0.04 \cdot \sin\left(\frac{\pi}{6} \cdot t + \frac{\pi}{6}\right)$$

$$b) v = \frac{dy}{dt} = 0.04 \cdot \frac{\pi}{6} \cdot \cos\left(\frac{\pi}{6} \cdot t + \frac{\pi}{6}\right) =$$

$$= 0.0209 \cdot \cos\left(\frac{\pi}{6} \cdot 5 + \frac{\pi}{6}\right) = \boxed{-0.0209 \frac{\text{m}}{\text{s}}}$$

$$a = \frac{dv}{dt} = -0.0209 \cdot \frac{\pi}{6} \cdot \sin\left(\frac{\pi}{6} \cdot t + \frac{\pi}{6}\right) =$$

$$= -0.0109 \cdot \sin\left(\frac{\pi}{6} \cdot 5 + \frac{\pi}{6}\right) = \boxed{0}$$

4) $E_m = 3 \cdot 10^{-5} \text{ J}$

$$F_{\max} = 1.5 \cdot 10^{-3} \text{ N}$$

¿A, ecuación?

$$T = 2 \text{ s}$$

$$E_m = \frac{1}{2} \cdot k \cdot A^2 ; F_{\max} = k \cdot A$$

$$\frac{E_m}{F_{\max}} = \frac{\frac{1}{2} \cdot k \cdot A^2}{k \cdot A} = \frac{A}{2} = \frac{3 \cdot 10^{-5}}{1.5 \cdot 10^{-3}}$$

$$A = \boxed{0.04 \text{ m}}$$

$$\omega = \frac{2 \cdot \pi}{T} = \frac{2 \cdot \pi}{2} = \pi \frac{\text{rad}}{\text{s}} ; x_0 = A \cdot \sin \varphi_0 \Rightarrow$$

$$0.02 = 0.04 \cdot \sin \varphi_0 \Rightarrow \sin \varphi_0 = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{6} \text{ rad}$$

$$x = 0.04 \cdot \sin\left(\pi \cdot t + \frac{\pi}{6}\right) \quad (118)$$

⑤ $x_1 = 0.04 \text{ m}$

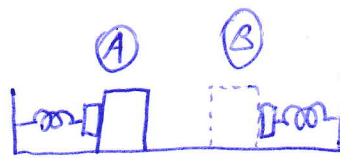
$m = 0.1 \text{ kg}$

$k_2 = 16 \frac{\text{N}}{\text{m}}$

$x_2 = 0.1 \text{ m}$

¿ k_1 ?

¿ f_2 ?



$E_{CA} + E_{PA} = E_{CB} + E_{PB}$

$\frac{1}{2} \cdot k_1 \cdot x_1^2 = \frac{1}{2} \cdot k_2 \cdot x_2^2$

$k_1 = k_2 \cdot \left(\frac{x_2}{x_1} \right)^2 = 16 \cdot \left(\frac{0.1}{0.04} \right)^2 = \boxed{100 \frac{\text{N}}{\text{m}}}$

$f_2 = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k_2}{m}} = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{16}{0.1}} = \boxed{2.01 \text{ Hz}}$

⑥ $m = 0.080 \text{ kg}$

$A = 0.05 \text{ m}$

¿Ecuación?

$E_{C_{\text{máx}}} = 2.5 \cdot 10^{-3} \text{ J}$

$t = 0 \Rightarrow x_0 = 0$

¿ E_M ?

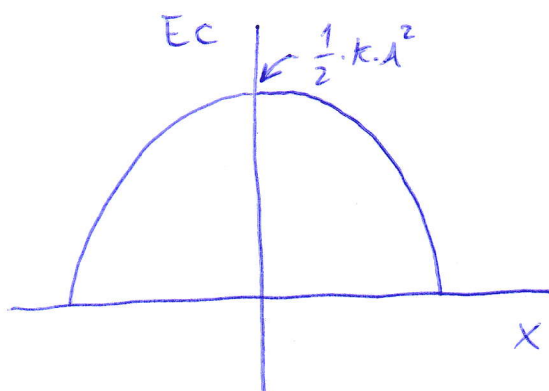
$x = A \cdot \sin(\omega \cdot t + \varphi_0)$

$E_{C_{\text{máx}}} = \frac{1}{2} \cdot k \cdot A^2 \Rightarrow k = \frac{2 \cdot E_{C_{\text{máx}}}}{A^2} =$

$= \frac{2 \cdot 2.5 \cdot 10^{-3}}{0.05^2} = 2 \frac{\text{N}}{\text{m}}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{0.08}} = 5 \frac{\text{rad}}{\text{s}}$

$x_0 = \sin \varphi_0 \Rightarrow \varphi_0 = 0 \Rightarrow \boxed{x = 0.05 \cdot \sin(5 \cdot t)}$



$E_M = E_{C_{\text{máx}}} =$

$= \boxed{2.5 \cdot 10^{-3} \text{ J}}$

$$\textcircled{7} \quad x_0 = 0.03 \text{ m}$$

$$v_0 = -0.12 \frac{\text{m}}{\text{s}}$$

$$T = 0.5 \text{ s}$$

¿Ecuación?

¿v, a? t = 4 s

¿v_{máx.}, a_{máx.}?

$$x = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$$\omega = \frac{2 \cdot \pi}{T} = \frac{2 \cdot \pi}{0.5} = 4 \cdot \pi \frac{\text{rad}}{\text{s}}$$

$$v = \frac{dx}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0)$$

$$\left. \begin{array}{l} v_0 = A \cdot \omega \cdot \cos \varphi_0 \\ x_0 = A \cdot \sin \varphi_0 \end{array} \right\} \begin{array}{l} -0.12 = A \cdot 4 \cdot \pi \cdot \cos \varphi_0 \\ 0.03 = A \cdot \sin \varphi_0 \end{array}$$

Dividiendo una entre otra:

$$\frac{0.03}{-0.12} = \frac{A \cdot \sin \varphi_0}{A \cdot 4 \cdot \pi \cdot \cos \varphi_0} \Rightarrow \tan \varphi_0 = \frac{0.03 \cdot 4 \cdot \pi}{-0.12} = -3.14 \Rightarrow$$

$$\Rightarrow \varphi_0 = -1.107 \text{ rad}$$

$$A = \left| \frac{x_0}{\sin \varphi_0} \right| = \left| \frac{0.03}{\sin(-1.107)} \right| = 0.0315 \text{ m}$$

$$\boxed{x = 0.0315 \cdot \sin(4 \cdot \pi \cdot t - 1.107)}$$

$$v = \frac{dx}{dt} = 0.0315 \cdot 4 \cdot \pi \cdot \cos(4 \cdot \pi \cdot 4 - 1.107) = 0.396 \cdot \cos(16 \cdot \pi - 1.107) = \boxed{0.121 \frac{\text{m}}{\text{s}}}$$

$$v_{\text{máx.}} = \boxed{0.396 \frac{\text{m}}{\text{s}}}$$

$$a = \frac{dv}{dt} = -0.396 \cdot 4 \cdot \pi \cdot \sin(4 \cdot \pi \cdot 4 - 1.107) = -4.98 \cdot \sin(16 \cdot \pi - 1.107) = \boxed{4.74 \frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{máx.}} = \boxed{4.98 \frac{\text{m}}{\text{s}^2}}$$

8) $A = 0.20 \text{ m}$

$t = 2 \text{ s}$

a) ¿Ecuación?

b) ¿ $v_{\text{máx.}}$, $a_{\text{máx.}}$?

c) ¿ t ?

a) $y = A \cdot \sin(\omega \cdot t + \varphi_0)$

$T = 4 \text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2 \cdot \pi}{4} = \frac{\pi}{2}$

$$y = 0.2 \cdot \sin\left(\frac{\pi}{2} \cdot t\right)$$

b) $v = \frac{dy}{dt} = 0.2 \cdot \frac{\pi}{2} \cdot \cos\left(\frac{\pi \cdot t}{2}\right) = 0.314 \cdot \cos\left(\frac{\pi \cdot t}{2}\right)$

$v_{\text{máx.}} = 0.314 \frac{\text{m}}{\text{s}}$

$a = \frac{dv}{dt} = -0.314 \cdot \frac{\pi}{2} \cdot \sin\left(\frac{\pi \cdot t}{2}\right) = -0.493 \cdot \sin\left(\frac{\pi \cdot t}{2}\right)$

$a_{\text{máx.}} = 0.493 \frac{\text{m}}{\text{s}^2}$

c) $v_{\text{máx.}} \Rightarrow \cos\left(\frac{\pi \cdot t}{2}\right) = 1 \Rightarrow \frac{\pi \cdot t}{2} = 0, \pi, 2\pi, \dots = n \cdot \pi \Rightarrow$

$\Rightarrow t = 2 \cdot n$

$a_{\text{máx.}} \Rightarrow \sin\left(\frac{\pi \cdot t}{2}\right) = 1 \Rightarrow \frac{\pi \cdot t}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots = (2n+1) \cdot \frac{\pi}{2} \Rightarrow$

$\Rightarrow \frac{\pi \cdot t}{2} = (2n+1) \cdot \frac{\pi}{2} \Rightarrow t = 2n+1$

9) $m = 0.3 \text{ kg}$

$k = 20 \frac{\text{N}}{\text{m}}$

$A = 0.05 \text{ m}$

a) $v, a?$

$y = 0.03 \text{ m}$

b) $v_{\text{máx.}}, a_{\text{máx.}}?$

a) $y = A \cdot \sin(\omega \cdot t + p_0)$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{0.3}} = 8.17 \frac{\text{rad}}{\text{s}}$

$y = 0.05 \cdot \sin(8.17 \cdot t)$

$y = 0.03 \Rightarrow 0.03 = 0.05 \cdot \sin(8.17 \cdot t) \Rightarrow$

$\Rightarrow \sin(8.17 \cdot t) = \frac{0.03}{0.05} = 0.6$

$8.17 \cdot t = 0.644 \Rightarrow t = \frac{0.644}{8.17} = 0.0788 \text{ s}$

$v = \frac{dy}{dt} = 0.05 \cdot 8.17 \cdot \cos(8.17 \cdot t) = 0.409 \cdot \cos(8.17 \cdot 0.0788) =$
 $= \boxed{0.327 \frac{\text{m}}{\text{s}}}$

$a = \frac{dv}{dt} = -0.409 \cdot 8.17 \cdot \sin(8.17 \cdot t) = -3.34 \cdot \sin(8.17 \cdot 0.0788) =$
 $= \boxed{-2 \frac{\text{m}}{\text{s}^2}}$

$v_{\text{máx.}} \Rightarrow \cos 0 = 1 \Rightarrow v_{\text{máx.}} = \boxed{0.409 \frac{\text{m}}{\text{s}}}$

$a_{\text{máx.}} \Rightarrow \sin 0 = 1 \Rightarrow a_{\text{máx.}} = \boxed{3.34 \frac{\text{m}}{\text{s}^2}}$

$$(10) \quad m = 0.04 \text{ kg}$$

$$x = 0.16 \text{ m}$$

$$a_{\max} = 0.36 \frac{\text{m}}{\text{s}^2}$$

$$T, f, k, E_M, v?$$

$$x = 0.02 \text{ m}$$

$$x = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$A = 0.08 \text{ m}$, pues es la mitad del recorrido de una oscilación completa.

$$v = \frac{dx}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0)$$

$$a = \frac{dv}{dt} = -A \cdot \omega^2 \cdot \sin(\omega \cdot t + \varphi_0)$$

$$a_{\max} = A \cdot \omega^2 \Rightarrow \omega = \sqrt{\frac{a_{\max}}{A}} = \sqrt{\frac{0.36}{0.08}} = 2.12 \frac{\text{rad}}{\text{s}}$$

$$T = \frac{2 \cdot \pi}{\omega} = \frac{2 \cdot \pi}{2.12} = \boxed{2.96 \text{ s}}$$

$$f = \frac{1}{T} = \frac{1}{2.96} = \boxed{0.338 \text{ Hz}}$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m \cdot \omega^2 = 0.04 \cdot 2.12^2 = \boxed{0.18 \frac{\text{N}}{\text{m}}}$$

$$E_M = \frac{1}{2} \cdot k \cdot A^2 = \frac{1}{2} \cdot 0.18 \cdot 0.08^2 = \boxed{5.76 \cdot 10^{-4} \text{ J}}$$

$$x = 0.02 \text{ m} \Rightarrow 0.02 = 0.08 \cdot \sin(\omega \cdot t + \varphi_0) \Rightarrow$$

$$\Rightarrow \sin(\omega \cdot t + \varphi_0) = \frac{0.02}{0.08} = 0.25 \Rightarrow \omega \cdot t + \varphi_0 = 0.253 \text{ rad}$$

$$v = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0) = 0.08 \cdot 2.12 \cdot \cos 0.253 = \boxed{0.164 \frac{\text{m}}{\text{s}}}$$

11) $l = 2 \text{ m}$

$T = 2.84 \text{ s}$

a) $\dot{r} g?$

b) $\dot{r} A?$

$v_{\text{máx.}} = 0.40 \frac{\text{m}}{\text{s}}$

c) $\dot{r} \text{ Ecuación?}$

a) $T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \cdot \frac{l}{g}$

$g = \frac{4\pi^2 \cdot l}{T^2} = \frac{4\pi^2 \cdot 2}{2.84^2} = \boxed{9.79 \frac{\text{m}}{\text{s}^2}}$

b) $x = A \cdot \sin(\omega \cdot t + \varphi_0)$

$v = \frac{dx}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0)$

$v_{\text{máx.}} = A \cdot \omega$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.84} = 2.21 \frac{\text{rad}}{\text{s}}$

$A = \frac{v_{\text{máx.}}}{\omega} = \frac{0.40}{2.21} = \boxed{0.181 \text{ m}}$

c) $x = A \cdot \sin(\omega \cdot t + \varphi_0) = \boxed{0.181 \cdot \sin(2.21 \cdot t)}$

$$\textcircled{12} \quad m = 0.2 \text{ kg} \quad \left| \quad a) \quad E_{p \max} = E_M = \frac{1}{2} \cdot k \cdot A^2 = \right.$$

$$k = 40 \frac{\text{N}}{\text{m}} \quad \left| \quad = \frac{1}{2} \cdot 40 \cdot 0.1^2 = \boxed{0.2 \text{ J}} \right.$$

$$A = 0.1 \text{ m}$$

$$b) \quad x = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{0.2}} = 14.1 \frac{\text{rad}}{\text{s}}$$

$$x = 0.03 \text{ m} \Rightarrow 0.03 = 0.10 \cdot \sin(\omega \cdot t + \varphi_0) \Rightarrow$$

$$\Rightarrow \sin(\omega \cdot t + \varphi_0) = \frac{0.03}{0.10} = 0.3 \Rightarrow \omega \cdot t + \varphi_0 = 0.305 \text{ rad}$$

$$v = \frac{dx}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi_0) = 0.1 \cdot 14.1 \cdot \cos 0.305 = 1.35 \frac{\text{m}}{\text{s}}$$

$$E_c = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot 0.2 \cdot 1.35^2 = \boxed{0.182 \text{ J}}$$

$$c) \quad x = 0.1 \cdot \sin(14.1 \cdot t)$$

$$v = \frac{dx}{dt} = 0.1 \cdot 14.1 \cdot \cos(14.1 \cdot t) = 1.41 \cdot \cos(14.1 \cdot t)$$

$$a = \frac{dv}{dt} = -1.41 \cdot 14.1 \cdot \sin(14.1 \cdot t) = -19.9 \cdot \sin(14.1 \cdot t)$$

$$a = 0.08 \Rightarrow 0.08 = -19.9 \cdot \sin(14.1 \cdot t) \Rightarrow$$

$$\Rightarrow \sin(14.1 \cdot t) = -\frac{0.08}{19.9} = -4.02 \cdot 10^{-3} \Rightarrow 14.1 \cdot t = 4.02 \cdot 10^{-3} \Rightarrow$$

$$\Rightarrow t = 2.85 \cdot 10^{-4} \text{ s} \Rightarrow v = 1.41 \cdot \cos(14.1 \cdot 2.85 \cdot 10^{-4}) = 1.41 \frac{\text{m}}{\text{s}}$$

$$E_c = \frac{1}{2} \cdot 0.2 \cdot 1.41^2 = \boxed{0.2 \text{ J}} \quad (12.5) \quad E_p = E_M - E_c = 0.2 - 0.2 = \boxed{0}$$

PROBLEMAS RESUELTOS DE CAMPOS

$$\textcircled{1} \quad M_M = \frac{M_T}{10}$$

$$R_M = \frac{R_T}{2}$$

$$c_M, P_M?$$

$$P_T = 700 \text{ N}$$

$$g_T = 9/8 \frac{\text{m}}{\text{s}^2}$$

$$P_T = m \cdot g_T \Rightarrow m = \frac{P_T}{g_T} =$$

$$= \frac{700}{9/8} = \boxed{71\frac{1}{4} \text{ kg}}$$

La masa es la misma en todas partes y en todos los planetas.

$$P_M = m \cdot g_M = m \cdot \frac{G \cdot M_M}{R_M^2} =$$

$$= m \cdot \frac{G \cdot \frac{M_T}{10}}{\left(\frac{R_T}{2}\right)^2} = m \cdot \frac{4 \cdot G \cdot M_T}{10 \cdot R_T^2} = \frac{2 \cdot m}{5} \cdot g_T =$$

$$= \frac{2 \cdot 71\frac{1}{4}}{5} \cdot 9/8 = \boxed{280 \text{ N}}$$

② $m = 100 \text{ kg}$

$h = 100 \text{ km} = 10^5 \text{ m}$

$\hat{v}_{\text{min}}?$

$\hat{v}_{\text{orb}}?$

$G = 667 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

$M_T = 598 \cdot 10^{24} \text{ kg}$

$R_T = 6370 \text{ km} = 637 \cdot 10^6 \text{ m}$

$E_{CA} + E_{PA} = E_{CB} + E_{PB}$

$\frac{1}{2} \cdot m \cdot v^2 - \frac{G \cdot M_T \cdot m}{R_T} = 0 - \frac{G \cdot M_T \cdot m}{R_T + h}$

$\frac{v^2}{2} - \frac{667 \cdot 10^{-11} \cdot 598 \cdot 10^{24}}{637 \cdot 10^6} =$

$= 0 - \frac{667 \cdot 10^{-11} \cdot 598 \cdot 10^{24}}{637 \cdot 10^6 + 10^5}$

$\frac{v^2}{2} - 626 \cdot 10^7 = -617 \cdot 10^7 \Rightarrow v = \boxed{1342 \frac{\text{m}}{\text{s}}}$

$F_G = F_c \Rightarrow \frac{G \cdot M \cdot m}{r^2} = \frac{m \cdot v^2}{r} \Rightarrow$

$\Rightarrow v = \sqrt{\frac{G \cdot M_T}{r}} = \sqrt{\frac{G \cdot M_T}{R_T + h}} =$

$= \sqrt{\frac{667 \cdot 10^{-11} \cdot 598 \cdot 10^{24}}{637 \cdot 10^6 + 10^5}} = \boxed{7852 \frac{\text{m}}{\text{s}}}$

$$\textcircled{3} \quad R_p = 2000 \text{ km} = 2 \cdot 10^6 \text{ m}$$

$$g_0 = 3 \frac{\text{m}}{\text{s}^2}$$

$$M_p?$$

$$v_e?$$

$$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$g_0 = \frac{G \cdot M_p}{R_p^2} \Rightarrow M_p = \frac{g_0 \cdot R_p^2}{G} =$$

$$= \frac{3 \cdot (2 \cdot 10^6)^2}{6.67 \cdot 10^{-11}} = \boxed{1.8 \cdot 10^{23} \text{ kg}}$$

$$E_{CA} + E_{PA} = E_{CA} + E_{PA}$$

$$\frac{1}{2} \cdot m \cdot v_e^2 - \frac{G \cdot M_p \cdot m}{R_p} = 0$$

$$v_e = \sqrt{\frac{2 \cdot G \cdot M_p}{R_p}} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 1.8 \cdot 10^{23}}{2 \cdot 10^6}} = \boxed{3465 \frac{\text{m}}{\text{s}}}$$

$$\textcircled{4} \quad m_1 = 2 \text{ kg} \rightarrow (0, 2)$$

$$m_2 = 3 \text{ kg} \rightarrow (0, -3)$$

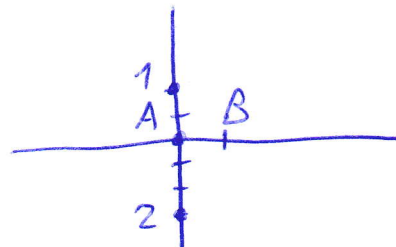
$$W_{AB}?$$

$$A(0, 0)$$

$$B(1, 0)$$

$$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$m_3 = 1 \text{ kg}$$



$$W_{AB} = -\Delta E_p = E_{pA} - E_{pB}$$

$$r_{1A} = 2 \text{ m}; \quad r_{2A} = 3 \text{ m}$$

$$r_{1B} = \sqrt{2^2 + 1^2} = 2.24 \text{ m}$$

$$r_{2B} = \sqrt{3^2 + 1^2} = 3.16 \text{ m}$$

$$E_{pA} = -\frac{G \cdot m_1 \cdot m_3}{r_{1A}} - \frac{G \cdot m_2 \cdot m_3}{r_{2A}} = -1.33 \cdot 10^{-10} \text{ J}$$

$$E_{pB} = -\frac{G \cdot m_1 \cdot m_3}{r_{1B}} - \frac{G \cdot m_2 \cdot m_3}{r_{2B}} = -1.23 \cdot 10^{-10} \text{ J}$$

$$W_{AB} = -\Delta E_p = -1.33 \cdot 10^{-10} + 1.23 \cdot 10^{-10} = \boxed{-10^{-11} \text{ J}}$$

(128)

$$⑤ \quad m = 2 \cdot 10^3 \text{ kg}$$

$$h = 5500 \text{ km}$$

$$a) \quad \hat{r} v_{orb.}?$$

$$b) \quad \hat{c} v?$$

$$v_0 = 0$$

$$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$M_T = 5.98 \cdot 10^{24} \text{ kg}$$

$$R_T = 6370 \text{ km}$$

$$a) \quad F_G = F_c$$

$$\frac{G \cdot M_T \cdot m}{r^2} = \frac{m \cdot v^2}{r}$$

$$v_{orb.} = \sqrt{\frac{G \cdot M_T}{r}} = \sqrt{\frac{G \cdot M_T}{R_T + h}} =$$

$$= \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{637 \cdot 10^6 + 55 \cdot 10^6}} =$$

$$= \boxed{5797 \frac{\text{m}}{\text{s}}}$$

$$b) \quad E_A^0 + E_{PA} = E_{cA} + E_{PB}$$

$$- \frac{G \cdot M_T \cdot m}{R_T + h} = \frac{1}{2} \cdot m \cdot v^2 - \frac{G \cdot M_T \cdot m}{R_T}$$

$$\frac{- 6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{637 \cdot 10^6 + 55 \cdot 10^6} = \frac{v^2}{2} - \frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{637 \cdot 10^6}$$

$$- 336 \cdot 10^7 = \frac{v^2}{2} - 626 \cdot 10^7$$

$$v = \boxed{7616 \frac{\text{m}}{\text{s}}}$$

⑥ $m = 100 \text{ kg}$

$v_{orb.} = 7.5 \cdot 10^3 \frac{\text{m}}{\text{s}}$

a) $\dot{r}$?

b) $\dot{E}_p$?

c) $\dot{E}_M$?

$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

$M_T = 5.98 \cdot 10^{24} \text{ kg}$

$R_T = 6370 \text{ km}$

a) $F_G = F_c$

$$\frac{G \cdot M_T \cdot m}{r^2} = \frac{m \cdot v^2}{r}$$

$$r = \frac{G \cdot M_T}{v^2} =$$

$$= \frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{(7.5 \cdot 10^3)^2} =$$

$$= \boxed{7.09 \cdot 10^6 \text{ m}}$$

b) $E_p = - \frac{G \cdot M_T \cdot m}{r} = \frac{-6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24} \cdot 100}{7.09 \cdot 10^6} =$

$$= \boxed{-5.63 \cdot 10^9 \text{ J}}$$

c) $E_M = E_c + E_p = \frac{1}{2} \cdot m \cdot v^2 - \frac{G \cdot M_T \cdot m}{r} =$

$$= \frac{1}{2} \cdot m \cdot \frac{G \cdot M_T}{r} - \frac{G \cdot M_T \cdot m}{r} = - \frac{G \cdot M_T \cdot m}{2 \cdot r} =$$

$$= - \frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24} \cdot 100}{2 \cdot 7.09 \cdot 10^6} = \boxed{-2.81 \cdot 10^9 \text{ J}}$$

$$\textcircled{7} R_M = 0.531 \cdot R_T$$

$$M_M = 0.107 \cdot M_T$$

$$r_{gM}?$$

$$r_t?$$

$$m = 1 \text{ kg}$$

$$h = 10 \text{ m}$$

$$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$M_T = 5.98 \cdot 10^{24} \text{ kg}$$

$$R_T = 6370 \text{ km}$$

$$g_M = \frac{G \cdot M_M}{R_M^2} =$$

$$= \frac{G \cdot 0.107 \cdot M_T}{(0.531 \cdot R_T)^2} =$$

$$= \frac{0.107}{(0.531)^2} \cdot \frac{G \cdot M_T}{R_T^2} =$$

$$= \frac{0.107 \cdot 6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{(0.531)^2 \cdot (6370 \cdot 10^3)^2} =$$

$$= \boxed{3.73 \frac{\text{m}}{\text{s}^2}}$$

$$e = v_y t + \frac{1}{2} \cdot g \cdot t^2 \Rightarrow t = \sqrt{\frac{2 \cdot e}{g}} =$$

$$= \sqrt{\frac{2 \cdot 10}{3.73}} = \boxed{2.32 \text{ s}}$$

$$\textcircled{8} \quad m = 0.15 \text{ kg}$$

$$h = 900 \text{ km} = 9 \cdot 10^5$$

$$\textcircled{a} F?$$

$$\textcircled{a} T?$$

$$G = 667 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

$$R_T = 637 \cdot 10^6 \text{ m}$$

$$M_T = 597 \cdot 10^{24} \text{ kg}$$

$$F = \frac{G \cdot M_T \cdot m}{r^2} =$$

$$= \frac{667 \cdot 10^{-11} \cdot 597 \cdot 10^{24} \cdot 0.15}{(637 \cdot 10^6 + 9 \cdot 10^5)^2} =$$

$$= \boxed{1.13 \text{ N}}$$

$$F_G = F_c \Rightarrow \frac{G \cdot M_T \cdot m}{r^2} = \frac{m \cdot v^2}{r} \Rightarrow v^2 = \frac{G \cdot M_T}{r}$$

$$v = \frac{2 \cdot \pi \cdot r}{T} \Rightarrow v^2 = \frac{4 \cdot \pi^2 \cdot r^2}{T^2}$$

$$\frac{G \cdot M_T}{r} = \frac{4 \cdot \pi^2 \cdot r^2}{T^2} \Rightarrow T^2 \cdot G \cdot M_T = 4 \cdot \pi^2 \cdot r^3 \Rightarrow$$

$$\Rightarrow T = \sqrt{\frac{4 \cdot \pi^2 \cdot r^3}{G \cdot M_T}} = \sqrt{\frac{4 \cdot \pi^2 \cdot (637 \cdot 10^6 + 9 \cdot 10^5)^3}{667 \cdot 10^{-11} \cdot 597 \cdot 10^{24}}} =$$

$$= \boxed{6172 \text{ s}}$$

9) $m_1 = m_2 = 50 \text{ kg}$

$A(-3, 0)$

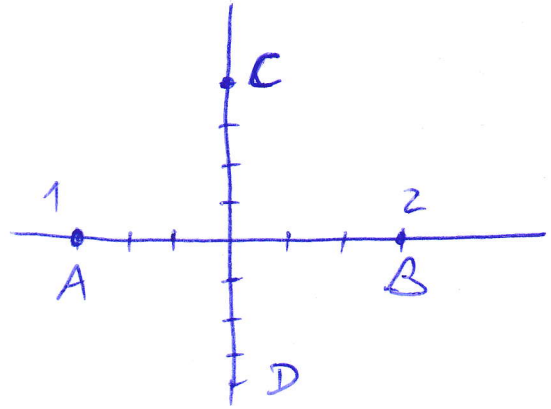
$B(3, 0)$

$m_3 = 30 \text{ kg}$

$C(0, 4) \text{ m}$

$D(0, -4) \text{ m}$

$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$



$W_{CD} = -\Delta E_p = E_{pC} - E_{pD}$

$r_{AC} = \sqrt{3^2 + 4^2} = 5 \text{ m} = r_{BC}$

$r_{AD} = \sqrt{3^2 + 4^2} = 5 \text{ m} = r_{BD}$

$$E_{pC} = -\frac{G \cdot m_1 \cdot m_3}{r_{AC}} - \frac{G \cdot m_2 \cdot m_3}{r_{BC}} = -G \cdot m_3 \cdot \left(\frac{m_1}{r_{AC}} + \frac{m_2}{r_{BC}} \right) =$$

$$= -6.67 \cdot 10^{-11} \cdot 30 \cdot \left(\frac{50}{5} + \frac{50}{5} \right) = -4 \cdot 10^{-8} \text{ J}$$

$$E_{pD} = \frac{-G \cdot m_1 \cdot m_3}{r_{AD}} - \frac{G \cdot m_2 \cdot m_3}{r_{BD}} = -G \cdot m_3 \cdot \left(\frac{m_1}{r_{AD}} + \frac{m_2}{r_{BD}} \right) =$$

$$= -6.67 \cdot 10^{-11} \cdot 30 \cdot \left(\frac{50}{5} + \frac{50}{5} \right) = -4 \cdot 10^{-8} \text{ J}$$

$$W_{CD} = E_{pC} - E_{pD} = -4 \cdot 10^{-8} + 4 \cdot 10^{-8} = \boxed{0 \text{ J}}$$

(10) $h_B = 20 \text{ km}$

$v_B = 18 \frac{\text{m}}{\text{s}}$

$v_A = ?$

$h_A = 30.000 \text{ km}$

$G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

$M_T = 5.97 \cdot 10^{24} \text{ kg}$

$R_T = 6.37 \cdot 10^6 \text{ m}$

$E_{CA} + E_{PA} = E_{CB} + E_{PB}$

$\frac{1}{2} \cdot m \cdot v_A^2 - \frac{G \cdot M_T \cdot m}{r_A} =$

$= \frac{1}{2} \cdot m \cdot v_B^2 - \frac{G \cdot M_T \cdot m}{r_B}$

$\frac{v_A^2}{2} - \frac{6.67 \cdot 10^{-11} \cdot 5.97 \cdot 10^{24}}{6.37 \cdot 10^6 + 3 \cdot 10^7} =$

$= \frac{18000^2}{2} - \frac{6.67 \cdot 10^{-11} \cdot 5.97 \cdot 10^{24}}{6.37 \cdot 10^6 + 2 \cdot 10^7}$

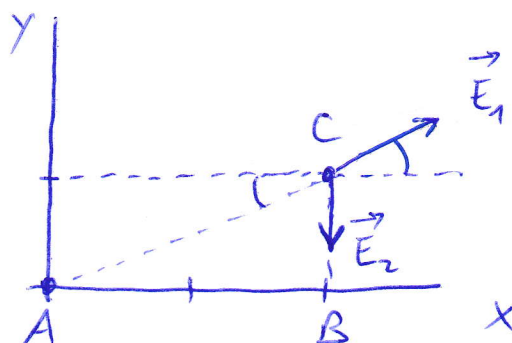
$\frac{v_A^2}{2} - 1.1 \cdot 10^7 = 1.62 \cdot 10^8 - 6.23 \cdot 10^7 \Rightarrow v_A = \boxed{1.49 \cdot 10^4 \frac{\text{m}}{\text{s}}}$

(11) $q_1 = 5 \cdot 10^{-6} \text{ C} \rightarrow A(0,0) \text{ m}$

$q_2 = -5 \cdot 10^{-6} \text{ C} \rightarrow B(2,0) \text{ m}$

$E? \rightarrow C(2,1) \text{ m}$

$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$



$E_1 = \frac{k \cdot q_1}{r_1^2} = \frac{9 \cdot 10^9 \cdot 5 \cdot 10^{-6}}{2^2 + 1^2} = 9000 \frac{\text{N}}{\text{C}}$

$E_2 = \frac{k \cdot q_2}{r_2^2} = \frac{9 \cdot 10^9 \cdot 5 \cdot 10^{-6}}{1^2} = 4.5 \cdot 10^4 \frac{\text{N}}{\text{C}}$

$\vec{E}_2 = -4.5 \cdot 10^4 \cdot \vec{j}$; $\vec{E}_1 = 9000 \cdot \frac{2}{\sqrt{5}} \cdot \vec{i} + 9000 \cdot \frac{1}{\sqrt{5}} \cdot \vec{j} =$

$= 8050 \cdot \vec{i} + 4025 \cdot \vec{j}$; $\vec{E}_T = \vec{E}_1 + \vec{E}_2 = 8050 \cdot \vec{i} - 4.1 \cdot 10^4 \cdot \vec{j}$

$E = \sqrt{E_x^2 + E_y^2} = (134) = \boxed{4.18 \cdot 10^4 \frac{\text{N}}{\text{C}}}$

12) $q_1 \rightarrow (0,0) \text{ m}$

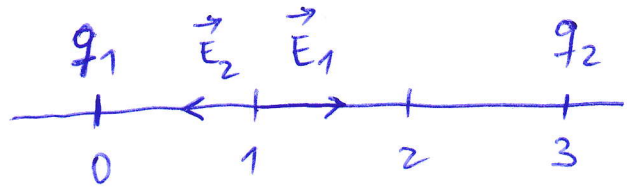
$q_2 \rightarrow (3,0) \text{ m}$

$E=0 \Rightarrow (1,0) \text{ m}$

$V = 9 \cdot 10^4 \text{ V}$

$q_1, q_2?$

$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$



$\vec{E} = \vec{E}_1 + \vec{E}_2 =$

$= \frac{k \cdot q_1}{r_1^2} \cdot \vec{i} - \frac{k \cdot q_2}{r_2^2} \cdot \vec{i} = 0 \Rightarrow$

$\Rightarrow \frac{k \cdot q_1}{r_1^2} = \frac{k \cdot q_2}{r_2^2} \Rightarrow$

$\Rightarrow \frac{q_1}{1^2} = \frac{q_2}{2^2} \Rightarrow q_2 = 4 \cdot q_1$

$V = V_1 + V_2 = \frac{k \cdot q_1}{d_0/2} + \frac{k \cdot q_2}{d_0/2} =$

$= \frac{9 \cdot 10^9 \cdot q_1 \cdot 2}{3} + \frac{9 \cdot 10^9 \cdot q_2 \cdot 2}{3} = 9 \cdot 10^4$

$q_1 + q_2 = \frac{9 \cdot 10^4 \cdot 3}{9 \cdot 10^9 \cdot 2} = 1.5 \cdot 10^{-5}$

$q_1 + q_2 = 1.5 \cdot 10^{-5} \quad \left. \begin{array}{l} q_1 + 4 \cdot q_1 = 1.5 \cdot 10^{-5} \\ q_2 = 4 \cdot q_1 \end{array} \right\} 5 \cdot q_1 = 1.5 \cdot 10^{-5} \Rightarrow q_1 = \boxed{3 \cdot 10^{-6} \text{ C}}$

$q_2 = 4 \cdot q_1 = 4 \cdot 3 \cdot 10^{-6} = \boxed{12 \cdot 10^{-6} \text{ C}}$

13) $q_1 = 8 \cdot 10^{-9} \text{ C} \rightarrow (0,0)$

$q_2 = -10^{-9} \text{ C} \rightarrow (3,0)$

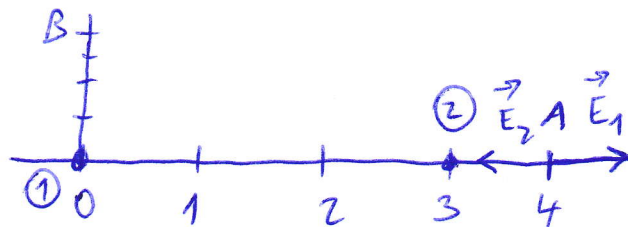
$\vec{r} \cdot E ? \rightarrow A(4,0) \text{ m}$

$\vec{r} \cdot W_{AB} ?$

$q = -2 \cdot 10^{-9} \text{ C}$

$B(0,4) \text{ m}$

$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$



$\vec{E}_1 = \frac{k \cdot q_1}{r_1^2} \cdot \vec{1} =$

$= \frac{9 \cdot 10^9 \cdot 8 \cdot 10^{-9}}{4^2} \cdot \vec{1} =$

$= 4.5 \cdot \vec{1}$

$\vec{E}_2 = \frac{-k \cdot q_2}{r_2^2} \cdot \vec{1} =$

$= \frac{-9 \cdot 10^9 \cdot 10^{-9}}{1^2} \cdot \vec{1} = -9 \cdot \vec{1}$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = 4.5 \cdot \vec{1} - 9 \cdot \vec{1} = -4.5 \cdot \vec{1}$

$E = \boxed{4.5 \frac{\text{N}}{\text{C}}}$

$W_{AB} = -\Delta E_P = E_{PA} - E_{PB}$

$E_{PA} = E_{PA1} + E_{PA2} = \frac{k \cdot q_1 \cdot q}{r_{A1}} + \frac{k \cdot q_2 \cdot q}{r_{A2}} =$

$= \frac{9 \cdot 10^9 \cdot 8 \cdot 10^{-9} \cdot (-2 \cdot 10^{-9})}{4} + \frac{9 \cdot 10^9 \cdot (-10^{-9}) \cdot (-2 \cdot 10^{-9})}{1} = -1.8 \cdot 10^{-8} \text{ J}$

$E_{PB} = E_{PB1} + E_{PB2} = \frac{k \cdot q_1 \cdot q}{r_{B1}} + \frac{k \cdot q_2 \cdot q}{r_{B2}} =$

$= \frac{9 \cdot 10^9 \cdot 8 \cdot 10^{-9} \cdot (-2 \cdot 10^{-9})}{4} + \frac{9 \cdot 10^9 \cdot (-10^{-9}) \cdot (-2 \cdot 10^{-9})}{5} =$

$= -3.24 \cdot 10^{-8} \text{ J} ; W_{AB} = -1.8 \cdot 10^{-8} + 3.24 \cdot 10^{-8} = \boxed{1.44 \cdot 10^{-8} \text{ J}}$

El signo positivo indica proceso espontáneo.

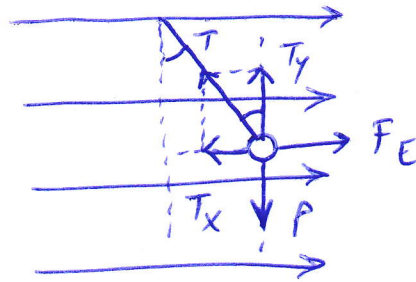
14) $m = 0.024 \text{ kg}$

$Q = 6 \cdot 10^{-3} \text{ C}$

$\alpha = 30^\circ$

Find E, T ?

$g = 9.8 \frac{\text{m}}{\text{s}^2}$



$$\left. \begin{aligned} \sum \vec{F}_x = 0 &\Rightarrow T_x = F_E \\ \sum \vec{F}_y = 0 &\Rightarrow T_y = P \end{aligned} \right\}$$

$$\left. \begin{aligned} T \cdot \sin \alpha &= Q \cdot E \\ T \cdot \cos \alpha &= m \cdot g \end{aligned} \right\}$$

$$\frac{T \cdot \sin \alpha}{T \cdot \cos \alpha} = \frac{Q \cdot E}{m \cdot g} ; \tan \alpha = \frac{Q \cdot E}{m \cdot g}$$

$$E = \frac{m \cdot g \cdot \tan \alpha}{Q} = \frac{0.024 \cdot 9.8 \cdot \tan 30^\circ}{6 \cdot 10^{-3}} = \boxed{22.6 \frac{\text{N}}{\text{C}}}$$

$$T = \frac{m \cdot g}{\cos \alpha} = \frac{0.024 \cdot 9.8}{\cos 30^\circ} = \boxed{0.272 \text{ N}}$$

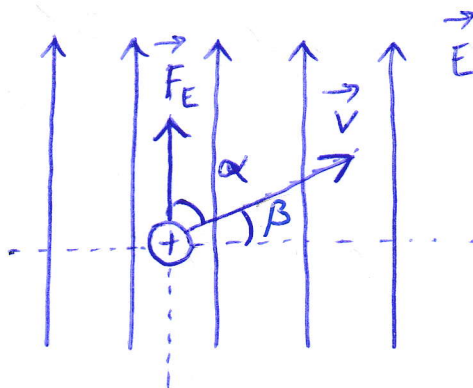
15) $Q = 2.5 \cdot 10^{-8} \text{ C}$

$E = 5 \cdot 10^4 \frac{\text{N}}{\text{C}}$

Find W_{AB} ?

$e = 0.5 \text{ m}$

$\beta = 30^\circ$



$$W_{AB} = F_E \cdot e \cdot \cos \alpha =$$

$$= Q \cdot E \cdot e \cdot \cos \alpha = 2.5 \cdot 10^{-8} \cdot 5 \cdot 10^4 \cdot 0.5 \cdot \cos 60^\circ =$$

$$= \boxed{3.12 \cdot 10^{-4} \text{ J}}$$

16 $Q_1 = 3 \cdot 10^{-6} \text{ C} \rightarrow (0,0) \text{ m}$

$Q_2 = -3 \cdot 10^{-6} \text{ C} \rightarrow (1,1) \text{ m}$

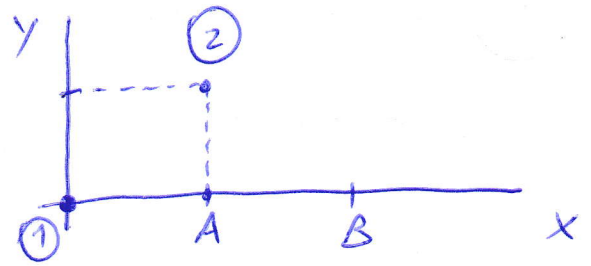
$W_{AB}?$

$Q_3 = 5 \cdot 10^{-6} \text{ C}$

$A(1,0) \text{ m}$

$B(2,0) \text{ m}$

$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$



$W_{AB} = -\Delta E_p = E_{pA} - E_{pB}$

$E_{pA} = E_{pA1} + E_{pA2} =$

$= \frac{k \cdot Q_1 \cdot Q_3}{r_{A1}} + \frac{k \cdot Q_2 \cdot Q_3}{r_{A2}} =$

$= 9 \cdot 10^9 \cdot 5 \cdot 10^{-6} \cdot \left(\frac{3 \cdot 10^{-6}}{1} + \frac{-3 \cdot 10^{-6}}{1} \right) = 0 \text{ J}$

$E_{pB} = \frac{k \cdot Q_1 \cdot Q_3}{r_{B1}} + \frac{k \cdot Q_2 \cdot Q_3}{r_{B2}} =$

$= 9 \cdot 10^9 \cdot 5 \cdot 10^{-6} \cdot \left(\frac{3 \cdot 10^{-6}}{2} - \frac{3 \cdot 10^{-6}}{\sqrt{2}} \right) = -0.028 \text{ J}$

$W_{AB} = E_{pA} - E_{pB} = 0 + 0.028 = \boxed{0.028 \text{ J}}$

El signo positivo indica que el proceso es espontáneo.

17) ¿W?

$$r_1 = 53 \cdot 10^{-11} \text{ m}$$

$$r_2 = 476 \cdot 10^{-10} \text{ m}$$

$$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$e = 1.6 \cdot 10^{-19} \text{ C}$$

$$W_{AB} = -\Delta E_P = E_{PA} - E_{PB} =$$

$$= \frac{k \cdot Q_1 \cdot Q_2}{r_1} - \frac{k \cdot Q_1 \cdot Q_2}{r_2} =$$

$$= k \cdot e \cdot (-e) \cdot \left(\frac{1}{r_1} - \frac{1}{r_2} \right) =$$

$$= -9 \cdot 10^9 \cdot (1.6 \cdot 10^{-19})^2 \cdot \left(\frac{1}{53 \cdot 10^{-11}} - \frac{1}{476 \cdot 10^{-10}} \right) =$$

$$= \boxed{-386 \cdot 10^{-18} \text{ J}}$$

El signo negativo indica que el proceso es no espontáneo.

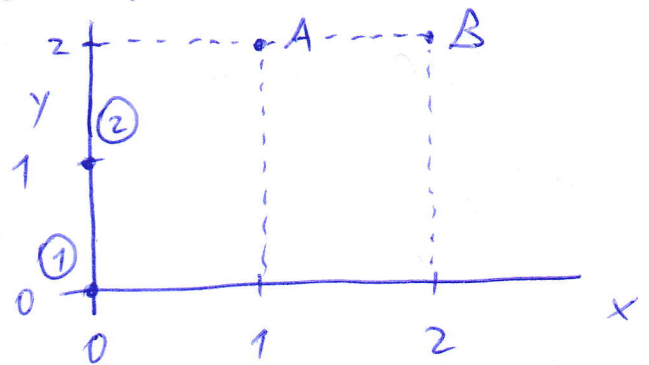
18) $Q_1 = 4 \cdot 10^{-9} \text{ C} \rightarrow (0,0) \text{ m}$

$Q_2 = -3 \cdot 10^{-9} \text{ C} \rightarrow (0,1) \text{ m}$

¿W_{AB}?

$Q_3 = 2 \cdot 10^{-9} \text{ C} \rightarrow A(1,2) \text{ m}$

$B(2,2) \text{ m}$



$$r_{A1} = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ m}$$

$$r_{A2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ m}$$

$$r_{B1} = \sqrt{2^2 + 2^2} = \sqrt{8} \text{ m}; \quad r_{B2} = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ m}$$

$$W_{AB} = -\Delta E_P = E_{PA} - E_{PB}$$

$$E_{PA} = E_{PA1} + E_{PA2} = \frac{9 \cdot 10^9 \cdot 4 \cdot 10^{-9} \cdot 2 \cdot 10^{-9}}{\sqrt{5}} + \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-9} \cdot 2 \cdot 10^{-9}}{\sqrt{2}} = -598 \cdot 10^{-9} \text{ J}$$

$$E_{PB} = E_{PB1} + E_{PB2} = \frac{9 \cdot 10^9 \cdot 4 \cdot 10^{-9} \cdot 2 \cdot 10^{-9}}{\sqrt{8}} + \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-9} \cdot 2 \cdot 10^{-9}}{\sqrt{5}} = 131 \cdot 10^{-9} \text{ J}$$

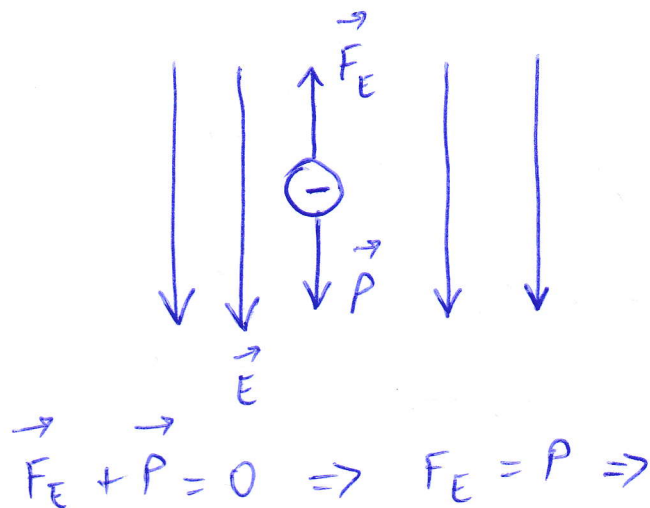
$$W_{AB} = \boxed{-729 \cdot 10^{-9} \text{ J}} \quad (139)$$

19) Q ?

$$m = 38 \cdot 10^{-3} \text{ kg}$$

$$E = 4500 \frac{\text{N}}{\text{C}}$$

$$g = 9.8 \frac{\text{m}}{\text{s}^2}$$



$$\Rightarrow Q \cdot E = m \cdot g \Rightarrow Q = \frac{m \cdot g}{E} = \frac{38 \cdot 10^{-3} \cdot 9.8}{4500} = \boxed{828 \cdot 10^{-6} \text{ C}}$$

20) $Q_1 = Q_2 = -3 \cdot 10^{-6} \text{ C}$

$$A(2, 5) \text{ m}$$

$$B(8, 2) \text{ m}$$

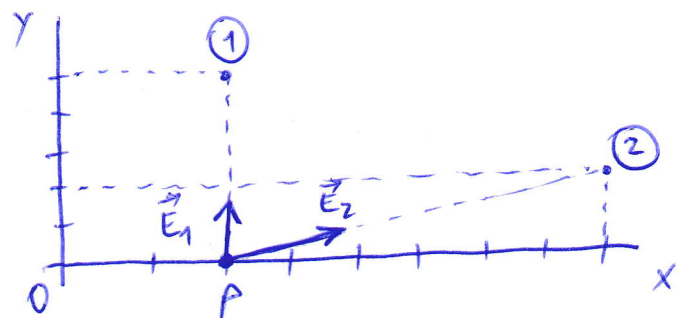
$$P(2, 0) \text{ m}$$

$$W_{P_0}?$$

$$Q_3 = 10^{-6} \text{ C}$$

$$O(0, 0) \text{ m}$$

$$k = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$



$$E_1 = \frac{k \cdot Q_1}{r_1^2} = \frac{9 \cdot 10^9 \cdot (+3) \cdot 10^{-6}}{5^2} = +1080 \frac{\text{N}}{\text{C}}$$

$$\vec{E}_1 = 1080 \cdot \vec{j}$$

$$E_2 = \frac{k \cdot Q_2}{r_2^2} = \frac{9 \cdot 10^9 \cdot 3 \cdot 10^{-6}}{2^2 + 6^2} = 675 \frac{\text{N}}{\text{C}}$$

$$\vec{E}_2 = E_2 \cdot \cos \alpha \cdot \vec{i} + E_2 \cdot \sin \alpha \cdot \vec{j} =$$

$$= 675 \cdot \frac{6}{\sqrt{40}} \cdot \vec{i} + 675 \cdot \frac{2}{\sqrt{40}} \cdot \vec{j} = 640 \cdot \vec{i} + 214 \cdot \vec{j}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 640 \cdot \vec{i} + (1080 + 214) \cdot \vec{j} =$$

$$= 640 \cdot \vec{i} + 1294 \cdot \vec{j}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{640^2 + 1294^2} = \boxed{1444 \frac{N}{C}}$$

$$W_{P_0} = -\Delta E_p = E_{P_P} - E_{P_0}$$

$$E_{P_P} = E_{P_{P1}} + E_{P_{P2}} = \frac{k \cdot Q_1 \cdot Q_3}{r_{P1}} + \frac{k \cdot Q_2 \cdot Q_3}{r_{P2}} =$$

$$= \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-6} \cdot 10^{-6}}{5} + \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-6} \cdot 10^{-6}}{\sqrt{40}} =$$

$$= -9'67 \cdot 10^{-3} \text{ J}$$

$$E_{P_0} = E_{P_{01}} + E_{P_{02}} = \frac{k \cdot Q_1 \cdot Q_3}{r_{01}} + \frac{k \cdot Q_2 \cdot Q_3}{r_{02}} =$$

$$= \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-6} \cdot 10^{-6}}{\sqrt{29}} + \frac{9 \cdot 10^9 \cdot (-3) \cdot 10^{-6} \cdot 10^{-6}}{\sqrt{68}} =$$

$$= -8'29 \cdot 10^{-3} \text{ J}$$

$$W_{P_0} = E_{P_P} - E_{P_0} = -9'67 \cdot 10^{-3} + 8'29 \cdot 10^{-3} =$$

$$= \boxed{-1'38 \cdot 10^{-3} \text{ J}}$$

El signo negativo indica que el proceso es no espontáneo.

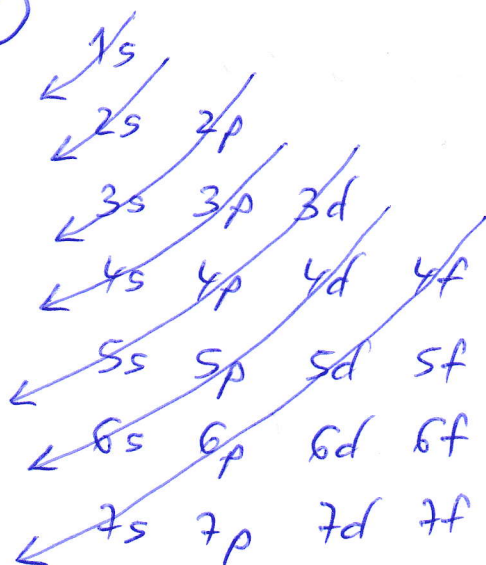
CUESTIONES RESUELTAS DE TEORÍA DE LA QUÍMICA

13

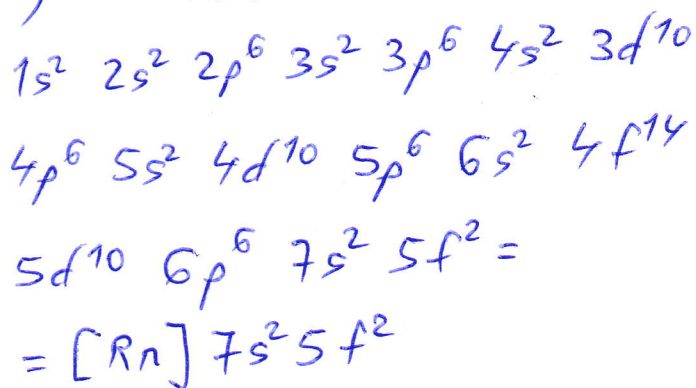
$$n=4 \left\{ \begin{array}{l} l=0 \rightarrow m=0 \rightarrow s=+1/2, -1/2 \\ l=1 \rightarrow m = \begin{cases} -1 \rightarrow \\ 0 \rightarrow \\ +1 \rightarrow \end{cases} \\ l=2 \rightarrow m = \begin{cases} -2 \rightarrow \\ -1 \rightarrow \\ 0 \rightarrow \\ +1 \rightarrow \\ +2 \rightarrow \end{cases} \\ l=3 \rightarrow m = \begin{cases} -3 \rightarrow \\ -2 \rightarrow \\ -1 \rightarrow \\ 0 \rightarrow \\ +1 \rightarrow \\ +2 \rightarrow \\ +3 \rightarrow \end{cases} \end{array} \right.$$

Ejemplos: $(4, 0, 0, +1/2)$, $(4, 2, -1, 1/2)$ y $(4, 3, 2, -1/2)$.

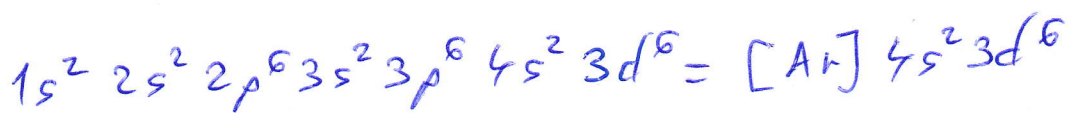
14

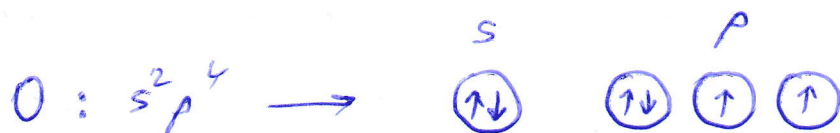
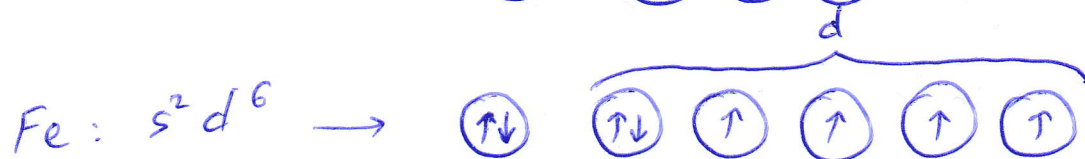
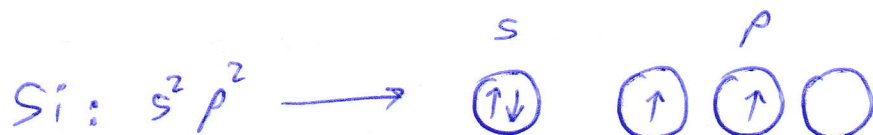
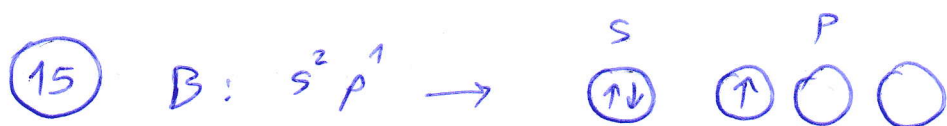


a) $Z=90$:



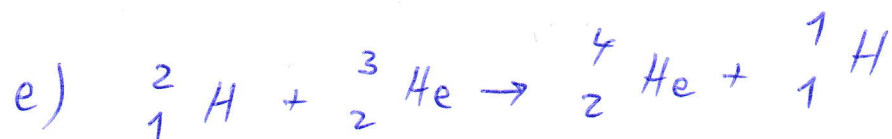
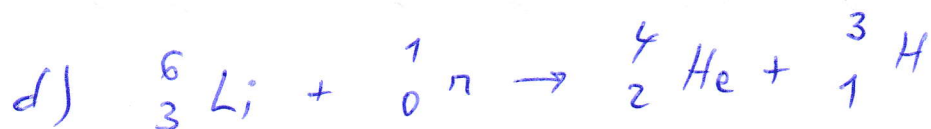
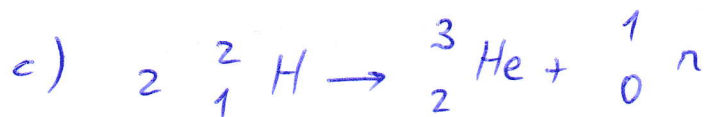
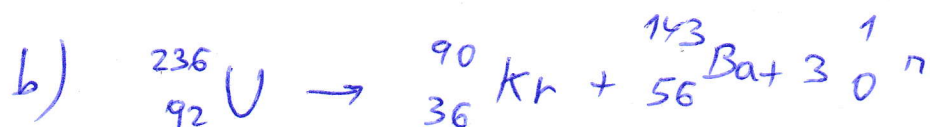
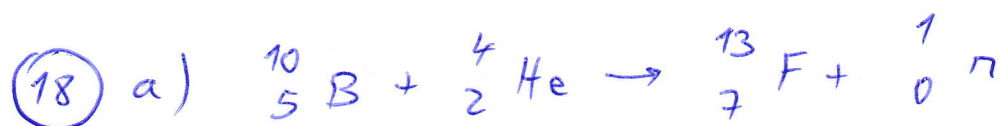
b) Fe:





16)

	$^{65}_{30}\text{Zn}$	$^{29}_{14}\text{Si}^{5-}$	$^{55}_{25}\text{Mn}^{2+}$	$^{108}_{47}\text{Ag}$	$^{128}_{52}\text{Te}^{6-}$
n	35	15	30	61	76
p	30	14	25	47	52
e ⁻	30	19	18	47	58



①9 - El enlace iónico: en que un átomo cede uno o varios electrones a otro; uno se convierte en anión y otro en catión y se atraen electrostáticamente.

- Enlace covalente: cada átomo aporta un electrón y el par de electrones gira en torno a los dos átomos.
- Enlace iónico: cada átomo de la estructura cristalina aporta uno o varios electrones que forman el gas electrónico y que gira en torno a todos los átomos.
- Fuerzas de van der Waals: el lado negativo de una molécula atrae al lado positivo de una molécula vecina.
- Enlace de hidrógeno: hay una atracción entre el hidrógeno y un átomo de su molécula y con un átomo electronegativo de una molécula vecina.

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Sustancia	Tipo enlace	Tipo sustancia	¿Soluble?	Dureza	PF y PE
Al_2O_3	covalente	covalente	No	Alta	Altos
KI	iónico	iónica	Sí	Alta	Altos
H_2	fuerzas van der Waals	molecular	No	Baja	Bajos
latón	metálico	metálica	No	Alta	Altos

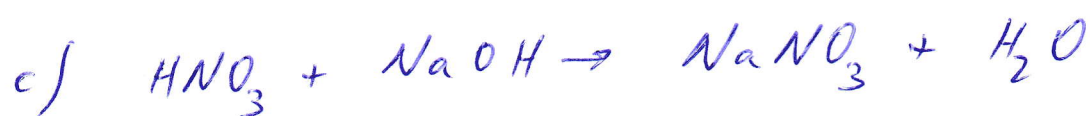
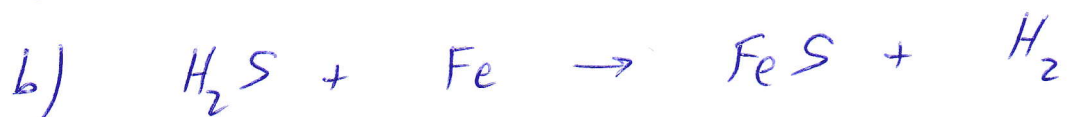
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Propiedad	S	L	G
Fuerzas interm.	Altas	Medias	Bajas
Distancia entre moléculas	Baja	Media	Alta
Densidad	Alta	Media	Baja
Viscosidad	Alta	Media-alta	Baja
¿Se comprimen?	No	No	Sí
Forma moverse	Vibración	Vibración Rotación Traslación	Vibración Rotación Traslación

(24) Por orden creciente de número atómico y de tal forma que en cada columna haya elementos de propiedades parecidas.

(26) $F > O > N > C > B > Be > Li$

El Ne no tiene electronegatividad porque no forma enlaces.



(29) a) No. Órbita es una trayectoria y orbital es una zona del espacio donde está el electrón. El concepto de órbita pertenece al modelo de Rutherford y al modelo de Bohr y el concepto de orbital al de la Mecánica Cuántica.

b) Incorrecto. Los electrones van ocupando los orbitales en orden creciente de energía (regla de $n+l$, diagrama de Mœller). Los electrones se extraen de los átomos en un orden distinto: los primeros en salir son los s y después los d o f .

c) Correcto. Según Rutherford, entre el núcleo y la corteza hay una gran distancia y no hay nada, luego la mayor parte de la materia es espacio vacío.

d) Falso. Tienen las mismas propiedades físicas pero distintas propiedades químicas.

e) Correcto. La dificultad de arrancar electrones y la facilidad de aceptar electrones.

(30) a) El volumen es una propiedad extensiva, cuantitativa y general.

b) La densidad es una propiedad intensiva, cuantitativa y específica.

c) El color es una propiedad intensiva, cualitativa y general.

(31) El tamaño de las partículas de soluto:

* suspensión: de 10^{-4} a 10^{-7} m.

* coloide: de 10^{-7} a 10^{-9} m.

* disolución: de 10^{-9} a 10^{-11} m.

(32) Se conservan: a, ~~k~~, d, g, h.

(33) Se conservan: nada.

(34) a) No están permitidos: (n, l, m, s)

(2, 1, 2, 1/2): m no puede ser mayor que l.

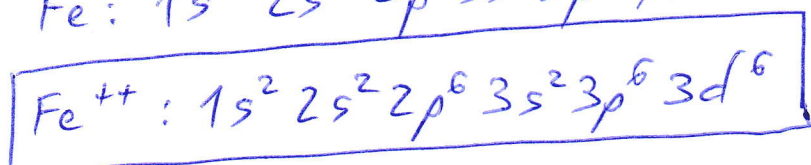
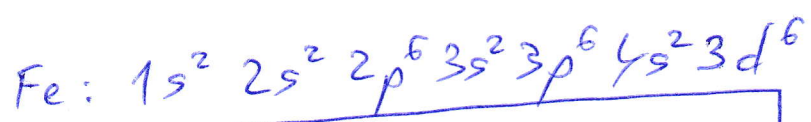
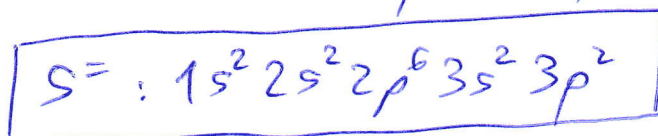
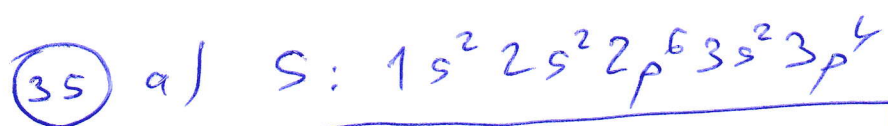
(2, 2, 1, -1/2): l no puede ser igual a n.

b) $l = 0, 1, 2, 3$
 s p d f

(3, 1, -1, 1/2) $\rightarrow$ 3p

(3, 2, -2, 1/2) $\rightarrow$ 3d

c) La energía de un orbital viene dada por la suma: $n+l$: $3p < 3d$.



b) Catión: K^+ , anión: Cl^- .

c) La energía de ionización mide la dificultad para arrancar un electrón de un átomo gaseoso. La segunda energía de ionización del magnesio es superior a la primera; esto significa que arrancar el segundo electrón es más difícil que el primero; esto es así porque la carga nuclear es la misma pero hay un electrón menos, es decir, los electrones que quedan están más atraídos por el núcleo.

36 a) $1s^2 2s^2 2p^6$: 2080 kJ/mol

$1s^2 2s^2 2p^6 3s^1$: 496 kJ/mol

La primera configuración es la de un gas noble, de las configuraciones más estables que hay. Arrancar un electrón es más difícil en una configuración de gas noble, luego la energía de ionización es mayor.

b) Es mayor la segunda energía de ionización del átomo de helio, pues es más difícil arrancar el electrón del He^+ que del H. Esto es así porque la carga nuclear es mayor.

37 a) $n=2, s=1/2$

Posibilidades: $(2, 0, 0, 1/2), (2, 1, 0, 1/2),$

$(2, 1, 1, 1/2), (2, 1, -1, 1/2).$

Cuatro electrones y dos orbitales. Un orbital viene dado por la pareja de los dos primeros números cuánticos.

b) $n=3, l=2.$

Posibilidades: $(3, 2, 2, 1/2), (3, 2, 2, -1/2),$

$(3, 2, 1, 1/2), (3, 2, 1, -1/2), (3, 2, 0, 1/2), (3, 2, 0, -1/2),$

$(3, 2, -1, 1/2), (3, 2, -1, -1/2), (3, 2, -2, 1/2),$

$(3, 2, -2, -1/2).$

Diez electrones y un orbital.

c) $n=4, l=3, m=-2$.

$(4, 3, -2, 1/2), (4, 3, -2, -1/2)$

Dos electrones y un orbital.

(38) a) Falso. Al tener mayor carga nuclear, los electrones están más atraídos por el núcleo y es más difícil arrancarlos.

b) Falso. Al tener mayor carga nuclear, tiene menor tamaño y los electrones de enlace están más atraídos por el núcleo.

c) Verdadero. Al tener menor carga nuclear, los electrones de valencia están menos atraídos por el núcleo y el átomo es más grande.

(39)

1s

2s 2p

3s 3p 3d

4s 4p 4d 4f

5s 5p 5d 5f

6s 6p 6d 6f

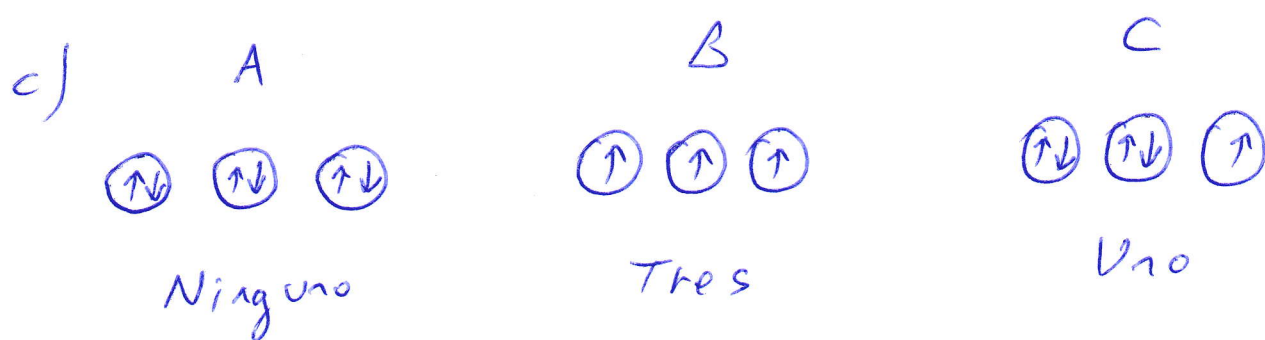
7s 7p 7d 7f

a) A: $1s^2 2s^2 2p^6 3s^2 3p^6$

B: $1s^2 2s^2 2p^6 3s^2 3p^6$
 $4s^2 3d^{10} 4p^3$

C: $1s^2 2s^2 2p^6 3s^2 3p^6$
 $4s^2 3d^{10} 4p^5$

b) El B porque tiene más tendencia a perder electrones.



d) B: $(4, 1, -1, 1/2)$
 $(4, 1, 0, 1/2)$
 $(4, 1, 1, 1/2)$

$l = 0, 1, 2, 3$
 s p d f

C: $(4, 1, 1, 1/2)$

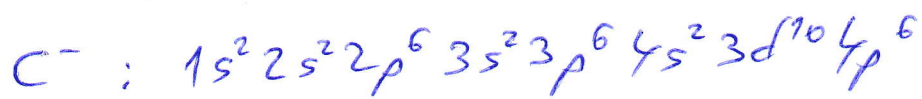
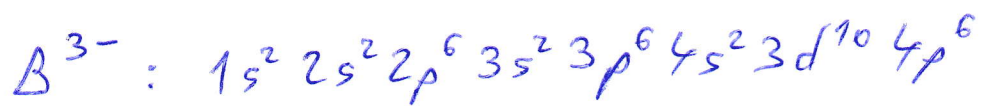
e) C es el de mayor electronegatividad porque A es un gas noble y porque C es más pequeño que B, luego atraerá más a los electrones de valencia.

f) El A porque es un gas noble y es más difícil arrancarle un electrón.

g) El B. B tiene mayor tamaño que A porque tiene un nivel electrónico más. B tiene mayor tamaño que C porque tiene menor carga nuclear y los electrones están menos atraídos.

h) El C. Porque tiene menor tamaño y el electrón está más atraído por el núcleo.

i) A: ninguno, pues es un gas noble.



j) El B^{3-} . Porque son especies isoelectrónicas y el B^{3-} tiene menor carga nuclear, luego atrae menos a los electrones.

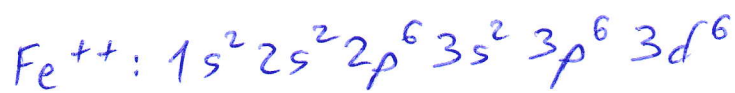
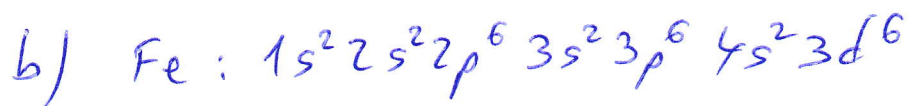
(40) a) $\text{A}^- > \text{A} > \text{A}^+$: este es el orden.

El anión tiene mayor tamaño porque tiene un electrón más y la carga nuclear es la misma. El catión tiene menor tamaño que el átomo neutro porque tiene un electrón menos y la carga nuclear es la misma.

b) Son aquellas que tienen el mismo número de electrones. Cl^- y K^+ son isoelectrónicos. N^{3-} , Mg^{++} y Al^{3+} son isoelectrónicos.

c) La de mayor tamaño será la de menor número atómico porque la carga nuclear es menor y los electrones están menos atraídos por el núcleo.

(41) a) Que se están llenando los orbitales d.



Los dos electrones apareados 4s son los dos primeros que se pierden.

c) $l = 0, 1, 2, 3$
s p d f

Porque para $l=3$ existen estos posibles orbitales:

$(n, 3, -3), (n, 3, 2), (n, 3, 1), (n, 3, 0), (n, 3, 1),$
 $(n, 3, 2)$ y $(n, 3, 3)$.

(42) a) Porque es más fácil arrancar un electrón de un átomo al ser los átomos cada vez más grandes y estar los electrones menos unidos al núcleo.

b) Porque es más difícil arrancar el segundo electrón que el primero pues la carga nuclear es la misma pero el número de electrones ha disminuido, luego estarán más atraídos por el núcleo.

c) Porque no se le puede arrancar un cuarto electrón, pues sólo tiene 3.

43) a) Sumando los exponentes y añadiendo los dos electrones que se le han quitado: $Z = 24$.

$$b) [M^{3+}] = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^3 = \boxed{[Ar] 3d^3}$$

$$c) \begin{matrix} l = 0, 1, 2, 3 \\ s \quad p \quad d \quad f \end{matrix}$$

3d significa: $n = 3$ y $l = 2$.

44) * Configuraciones electrónicas:

$$A: 1s^2 2s^2 2p^5$$

$$B: 1s^2 2s^2 2p^6 3s^2 3p^4$$

$$C: 1s^2 2s^2 2p^6 3s^2 3p^5$$

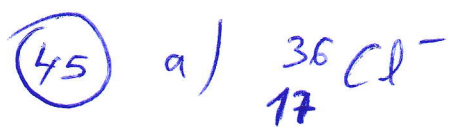
$$D: 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$$

$$E: 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$$

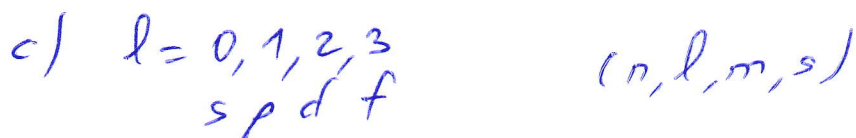
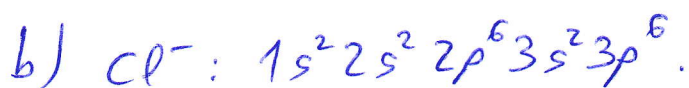
a) El A. Porque es el más pequeño y los electrones de enlace están más atraídos por el núcleo, que está más cercano.

b) El D. Porque es de mayor tamaño y es más fácil arrancarle un electrón.

c) El E. Porque es un alcalinotérreo, tiene tendencia a perder electrones y, al hacerlo, su configuración es la de gas noble, muy estable.

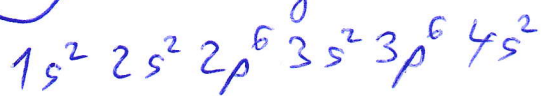


Protones: 17, electrones: $17+1=18$, neutrones: $36-17=19$



$$(3, 1, 1, 1/2)$$

46 * Configuración electrónica:

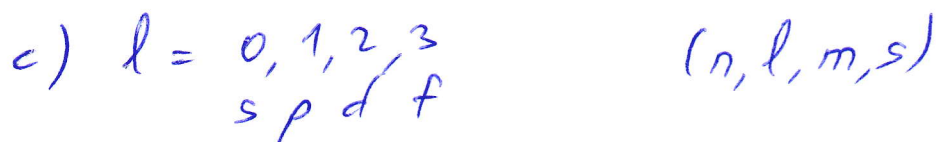


a) Grupo: alcalinotérreos, pues el último orbital tiene dos electrones 4s.

Periodo: cuarto. El 4 de 4s indica el periodo.

Otro del mismo grupo: magnesio.

b) +2. Pues tiene tendencia a perder dos electrones y así adquirir la configuración de gas noble.



$$4s^2 \rightarrow (4, 0, 0, -1/2)$$

47) * Configuraciones electrónicas:

A: $1s^2$

B: $1s^2 2s^2 2p^5$

C: $1s^2 2s^2 2p^6 3s^1$

D: $1s^2 2s^2 2p^6 3s^2$

E: $1s^2 2s^2 2p^6 3s^2 3p^1$

a) El A, que es el He.

b) El C, pues su configuración externa es ns^1 .

c) El B, pues es el más pequeño y los electrones de enlace están más atraídos por un núcleo más pequeño.

d) AB $\rightarrow$ No existiría, pues el helio no forma enlaces al tener una configuración muy estable.

CD $\rightarrow$ No existiría un compuesto como tal, pues los dos son metales y no forman un compuesto químico.

(48) a) La A. Principio de exclusión: en un átomo no puede haber dos electrones con los cuatro números cuánticos iguales.

b) La D. Si hay dos electrones en el orbital $2p$, tienden a estar lo más despareados posible, de esta manera: $2p_x^1 2p_y^1 2p_z^0$.

c) La C. Diagrama de orbitales externos:



(49) a) l va desde 0 a $n-1$, luego los subniveles de energía serían: $l = 0, 1, 2, 3$.

b) $l = 0, 1, 2, 3$
 $s \quad p \quad d \quad f$

c) Para $n = 5$, $l = 0, 1, 2, 3$ y 4.

La energía de un orbital viene dada por la suma: $n + l$.

$$5s < 4d \rightarrow n + l = 4 + 2 = 6$$

↓

$$n + l = 5 + 0 = 5$$

(50) a) Falso. Es 18. El número máximo de electrones viene dado por: $2 \cdot n^2 = 2 \cdot 3^2 = 2 \cdot 9 = 18$.

b) Falso. Pueden haber 6, pues existen tres tipos de orbitales p: p_x , p_y y p_z y en cada uno puede haber dos electrones como máximo.

c) Falso. Habrá 2.

Diagrama de orbitales: $\uparrow\downarrow \uparrow \uparrow \uparrow$

d) Falso. Tiene menor tamaño porque tiene un nivel electrónico menos.

e) Verdadero. Al ser más pequeño el O, atrae más a los electrones de enlace.

f) Falso. La tiene menor porque es más fácil arrancarle electrones y adquirir la configuración de gas noble.

g) Falso. Es según el número atómico creciente.

h) Falso. El B es más electronegativo al ser más pequeño y atraer más a los electrones de enlace.

i) Verdadero. Al ser más pequeño el Si, sus electrones están más atraídos por el núcleo y es más difícil arrancarle un electrón.

j) Verdadero. Ambos tienen el mismo número de niveles electrónicos pero el C tiene menor carga nuclear y atrae menos a los electrones.

k) Verdadero. Al ser más pequeño el O, tiene más tendencia a adquirir un electrón.

l) Verdadero. Al ser más pequeño el As, tiene más tendencia a captar un electrón.

m) Falso. Los valores posibles de l son de 0 a $n-1$, es decir: 0 y 1.

$l = 0, 1, 2, 3$ Sólo son posibles un orbital s
s p d f y tres orbitales p.

n) Falso. n vale 3 pues es el número que aparece en el orbital 3p.

ñ) Verdadero. (n, l, m, s)

$$n=4 \left\{ \begin{array}{l} l=0 \rightarrow m=0 \\ l=1 \rightarrow m=\begin{matrix} -1 \\ 0 \\ 1 \end{matrix} \\ l=2 \rightarrow m=\begin{matrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{matrix} \rightarrow s=\pm 1/2 \\ l=3 \rightarrow m=\begin{matrix} -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \rightarrow s=\pm 1/2 \end{array} \right.$$

Los electrones serían:

$(4, 2, -2, 1/2)$
 $(4, 2, -2, -1/2)$
 $(4, 3, -2, 1/2)$
 $(4, 3, -2, -1/2)$

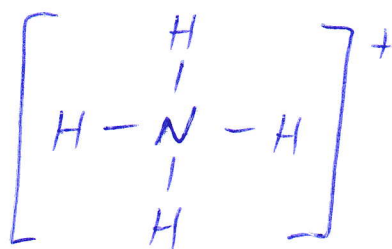
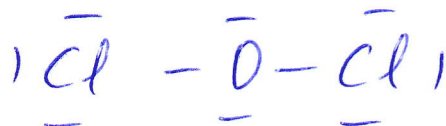
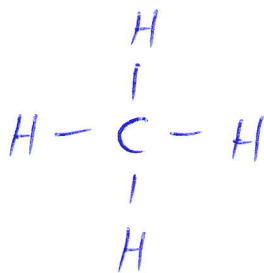
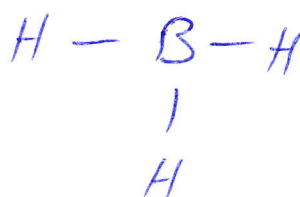
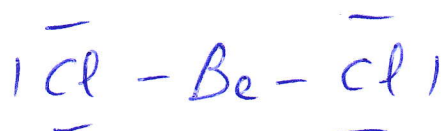
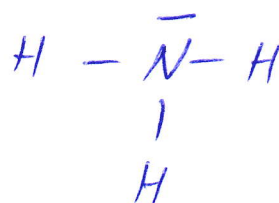
o) Falso. El número máximo de electrones viene dado por: $2 \cdot n^2 = 2 \cdot 3^2 = 18$.

p) Falso. Habrá 3. Diagrama de orbitales:



Según el principio de máxima multiplicidad, los electrones tenderán a estar lo más despareados posible.

q) Verdadero. Orbital s significa $l=0$. Sólo hay un orbital $4s$ y en cada orbital pueden haber dos electrones.



(52) a) $U_{LiF} > U_{NaF} > U_{KF}$. La energía reticular es proporcional a: $\frac{Q_1 \cdot Q_2}{d_0^2}$. A menor radio iónico, menor distancia entre iones (d_0) y mayor energía reticular (U).

b) $TF_{LiF} > TF_{NaF} > TF_{KF}$. Cuando una sustancia se funde, se rompe la red cristalina. A mayor energía reticular, más difícil es romper la red y la temperatura de fusión es más alta.

(53) a) El CaO , pues es una sustancia iónica. En estado sólido, los electrones están localizados alrededor de cada ion. En estado líquido, los iones se mueven con facilidad.

b) El I_2 . Es una sustancia molecular. Una sustancia que sublima tiene bajas fuerzas intermoleculares, como el I_2 , que tiene fuerzas de van der Waals.

c) El Cu , pues es una sustancia metálica. El enlace metálico le confiere dureza, ductilidad y maleabilidad.

(54) a) $U_{NaBr} > U_{KBr} > U_{CsBr}$. La energía reticular es proporcional a: $\frac{Q_{catión} \cdot Q_{anión}}{d_0^2}$.

Cuanto menor sea la distancia entre iones, d_0 , mayor será la energía reticular. A menor radio atómico, menor radio iónico y menor d_0 :

$$r_{Na} < r_K < r_{Cs}.$$

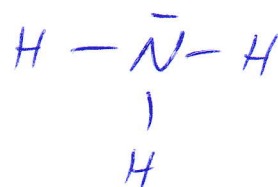
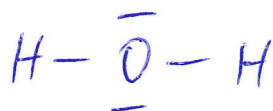
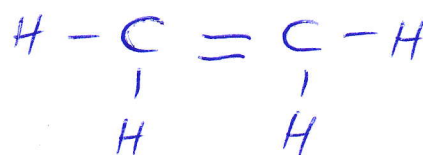
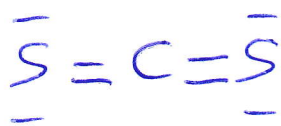
b) El NaBr. Cuando una sustancia iónica se disuelve, se rompe su red cristalina por las atracciones electrostáticas con el disolvente. Cuanto mayor sea la energía reticular, más difícil será romper la red y disolver el compuesto.

(55) a) Enlace iónico. Debido a la gran diferencia de electronegatividades, el Na y el Cl forman enlace iónico. El Na le da un e^- al Cl.

b) Enlace de hidrógeno. Las moléculas de agua están unidas por enlace de hidrógeno: el hidrógeno se une con el O de su molécula y con el O de una molécula vecina.

c) Fuerzas de van der Waals. Cuando se vaporiza una sustancia molecular, se rompen las fuerzas intermoleculares.

(56) * Estructuras de Lewis:



a) F_2 , H_2O y NH_3 .

b) CS_2 y C_2H_4 .

c) C_2H_2 .

(57) a) Diamante: enlace covalente pues los átomos de C se unen entre sí formando una estructura muy compacta.

Fe: enlace iónico: pues cada átomo de hierro cede algunos electrones que giran en torno a toda la red cristalina.

HBr: enlace de hidrógeno, pues el hidrógeno está unido a un átomo muy electronegativo.

NaBr: enlace iónico, pues entre el Na y el Br hay una gran diferencia de electronegatividades.

Br_2 : fuerzas de van der Waals, pues la molécula es apolar.

b) El Fe, pues el enlace metálico es un enlace fuerte en el hierro.

c) Son insolubles el diamante, el Fe y el Br_2 , pues no interaccionan apenas con el disolvente.

(58) a) Enlaces de hidrógeno. Cuando una sustancia molecular se funde, se rompen algunas fuerzas intermoleculares que, en el agua, son enlaces de hidrógeno.

b) Enlace iónico. Debido a la gran diferencia de electronegatividades entre el Na y el Cl, forman enlace iónico. Cuando se disuelve, se rompe la red cristalina.

c) Fuerzas de van der Waals. Cuando una sustancia molecular sublima, se rompen las fuerzas intermoleculares. El I_2 tiene FVW por ser una molécula apolar.

d) Enlace covalente. Los átomos de C se unen entre sí formando una estructura muy compacta.

e) Enlace metálico. Cada átomo de cobre aporta varios electrones para formar el gas electrónico.

- (59) a) Falso. Al ser un compuesto iónico, cuando se disuelve los iones tienen libertad de movimiento y se conduce la corriente eléctrica.
- b) Verdadero. La estructura tetraédrica del SiH_4 lo hace apolar e insoluble en agua. El NaCl es soluble porque es una sustancia iónica y sus iones quedan libres en disolución.
- c) Falso. Es bajo por ser una sustancia molecular con fuerzas de van der Waals.
- d) Verdadero. La geometría triangular plana anula la polaridad de la molécula.
- e) Falso. Al tener el Na menor radio iónico que el Cs, la energía reticular del NaCl es mayor que la del CsCl y es más difícil fundirlo.
- f) Verdadero. En el NaCl(s) , los electrones están localizados alrededor de cada ion. En el cobre, los electrones están deslocalizados en la nube electrónica.
- g) Verdadero. Cuando la diferencia de electronegatividades entre los elementos está entre 1 y 1'6, el enlace es covalente polar.

h) Verdadero. Cuando están fundidos o disueltos, los iones quedan libres y se transmite la electricidad.

i) Verdadero. Esto se debe a que el O es el más electronegativo del grupo y su enlace de hidrógeno es más fuerte.

j) Verdadero. El O tiene dos electrones menos que el Ne, al tener Z dos unidades inferior. Al ganar dos electrones, se convierte en O^{2-} y tiene los mismos electrones y la misma configuración que el Ne.

k) Falso. La energía de ionización del gas noble es superior porque su configuración electrónica es más estable.

l) Falso. Al adquirir dos electrones, el O cambia su número de electrones, no de protones.

m) Verdadero. El HCl se disocia en agua en iones que conducen la electricidad.

n) Falso. Las configuraciones $1s^2$ o $ns^2 np^6$ de los gases nobles son muy estables y por ello apenas tienen tendencia a reaccionar.

n) Falso. Es triangular plana.

o) Falso. Es la energía necesaria para formar una red cristalina a partir de sus iones en estado gaseoso.

p) Falso. Al ser el Li más pequeño que el K y el F más pequeño que el Br, las distancias entre iones son así: $d_{LiF} < d_{KBr}$, luego $U_{LiF} > U_{KBr}$ y es más difícil romper la red cristalina y disolver al LiF.

q) Falso. Al ser el Mg más pequeño que el Ca y el O más pequeño que el S, las distancias entre iones serían: $d_{MgO} < d_{CaS}$, luego $U_{MgO} > U_{CaS}$ y es más difícil romper la red cristalina y disolver al MgO.

r) Falso. Los electrones de distintos elementos son exactamente iguales. Son buenos conductores porque sus electrones forman un gas electrónico que tiene libertad de movimiento.

s) Verdadero. Esto es así porque el etanol interactúa con el agua formando enlaces de hidrógeno y el etano no lo hace porque es apolar, mientras que el agua es polar.

- t) Verdadero. Esto es así porque, a medida que crece la masa atómica, crece la intensidad de las fuerzas de van der Waals y los puntos de fusión y de ebullición son cada vez más altos.
- u) Falso. En estado sólido, los electrones están localizados en los compuestos iónicos y no conducen la electricidad.
- v) Verdadero. Las tres son sustancias moleculares, pero el H_2 y el I_2 son apolares y no interaccionan con el agua; el HI es polar y sí interacciona con la molécula polar del agua.
- w) Verdadero. Esto es debido a que la geometría triangular plana de la molécula causa la polaridad.
- x) Falso. Al ser una sustancia iónica, en estado sólido sus electrones están localizados y no conducen la electricidad.
- y) Verdadero. Al ser una sustancia iónica, en agua se rompe su estructura cristalina y los iones se rodean de moléculas de agua.
- z) Verdadero. Las fuerzas intermoleculares del NH_3 (enlace de hidrógeno) son más intensas que las del CH_4 (fuerzas de van der Waals).

- 60) a) Falso. Ambos tienen fuerzas de van der Waals, que crecen con la masa molecular.
- b) Verdadero. Las fuerzas intermoleculares del CH_3OH (enlace de hidrógeno) son más fuertes que las del CH_4 (fuerzas de van der Waals).
- c) Verdadero. El enlace iónico del KCl es más fuerte que las fuerzas de van der Waals del Cl_2 .
- d) Falso. El CCl_4 es una sustancia molecular apolar y es insoluble en una molecular apolar como el agua. El KCl es una sustancia iónica y es soluble porque sus iones se rodean de moléculas de agua y las atraen.
- e) Falso. Al ser el O más pequeño que el S, las distancias entre iones son: $d_{\text{CaO}} < d_{\text{CaS}}$, luego: $U_{\text{CaO}} > U_{\text{CaS}}$. Luego es más difícil romper la red cristalina y disolver el CaO .
- f) Falso. Es más difícil romper las fuerzas intermoleculares en el NH_3 (enlace de hidrógeno) que en el CH_4 (fuerzas de van der Waals).

g) Verdadero. Las fuerzas entre partículas son mucho mayores en el KCl (enlace iónico) que en el Cl_2 (fuerzas de van der Waals).

h) Verdadero. El CH_4 es una sustancia molecular apolar y es insoluble en la molécula polar del agua. El KCl es soluble porque en agua se rompe su red cristalina y los iones se solvatan (se rodean de moléculas de agua y las atraen electrostáticamente).

i) Falso. Las fuerzas intermoleculares del etanol (enlace de hidrógeno) son más fuertes que las del etano (fuerzas de van der Waals).

j) Verdadero. La geometría tetraédrica de la molécula anula su polaridad.

k) Falso. Al ser el Mg más pequeño que el Ba , se cumple que las distancias entre iones son: $d_{MgO} < d_{BaO}$, luego: $U_{MgO} > U_{BaO}$. Luego es más difícil romper la red cristalina y disolver a MgO .

l) Falso. Normalmente no, pues sus electrones están localizados.

m) Verdadero. Esto es así porque el enlace covalente es fuerte y hay enlaces covalentes uniendo todos los átomos de la red cristalina.

n) Verdadero. Cuando se disuelve un compuesto iónico, la red cristalina se rompe, los iones quedan libres y se conduce la corriente eléctrica.

ñ) Verdadero. El agua pura se descompone en H^+ y OH^- , que podrían conducir la electricidad, pero su concentración es pequetísima: $10^{-7} M$.

o) Falso. En estado sólido, las sustancias iónicas tienen electrones localizados y no conducen la electricidad.

p) Verdadero. En disolución, las sustancias iónicas rompen su red cristalina, los iones quedan libres en disolución y conducen la electricidad.

q) Verdadero. Aunque los enlaces $Be-H$ y $H-S$ son polares, la geometría molecular lineal del $H-Be-H$ anula la polaridad pero no así la geometría angular del H_2S :



r) Falso. En el enlace covalente, ya sea polar o apolar, cada átomo da un electrón y los dos electrones giran alrededor de los dos átomos. En el enlace covalente dativo o coordinado, es un átomo el que aporta los dos electrones.

s) Falso. El Na tiene menor tamaño que el Cs, luego las distancias entre iones serían: $d_{\text{NaI}} < d_{\text{CsI}}$, luego $U_{\text{NaI}} > U_{\text{CsI}}$. Al tener mayor energía reticular, es más difícil romper la red cristalina del NaI y disolverlo.

t) Falso. El NaCl es una sustancia iónica y las sustancias iónicas no forman moléculas, pues el enlace iónico une todos los iones de la red cristalina.

PROBLEMAS RESUELTOS DE CÁLCULOS QUÍMICOS

① * Abundancia isotópica del Plomo:

$$\text{Porcentaje} = 100 - 2 - 28.2 - 57.8 = 12 \%$$

* Masa atómica del plomo:

$$A = \sum \frac{\text{Porcentaje}_i \cdot A_i}{100} =$$

$$= \frac{2 \cdot 204}{100} + \frac{28.2 \cdot 207}{100} + \frac{57.8 \cdot 208}{100} + \frac{12 \cdot 209}{100} = \boxed{207.76}$$

② a)

$$E = h \cdot \frac{c}{\lambda} = 6.62 \cdot 10^{-34} \cdot \frac{3 \cdot 10^8}{500 \cdot 10^{-9}} = \boxed{3.97 \cdot 10^{-19} \text{ J}}$$

$$b) E = h \cdot f = 6.62 \cdot 10^{-34} \cdot 670 \cdot 10^{12} = \boxed{4.44 \cdot 10^{-19} \text{ J}}$$

$$\textcircled{3} \quad \frac{1}{\lambda} = R \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = 10967758 \cdot \left(\frac{1}{3^2} - \frac{1}{5^2} \right) =$$

$$= 78 \cdot 10^5 \Rightarrow \lambda = \frac{1}{78 \cdot 10^5} = 1.28 \cdot 10^{-6} \text{ m}$$

$$E = h \cdot \frac{c}{\lambda} = 6.62 \cdot 10^{-34} \cdot \frac{3 \cdot 10^8}{1.28 \cdot 10^{-6}} = \boxed{1.55 \cdot 10^{-19} \text{ J}}$$

$$④ \quad v = \frac{c}{2} = \frac{3 \cdot 10^8}{2} = 1,5 \cdot 10^8 \frac{m}{s}$$

$$\lambda = \frac{h}{m \cdot v} = \frac{6,62 \cdot 10^{-34}}{9,1 \cdot 10^{-31} \cdot 1,5 \cdot 10^8} = \boxed{4,85 \cdot 10^{-12} m}$$

$$⑤ \quad E_c = 12 eV = 12 \cdot 1,6 \cdot 10^{-19} = 1,92 \cdot 10^{-18} J$$

$$E_c = \frac{1}{2} \cdot m \cdot v^2 \Rightarrow v = \sqrt{\frac{2 \cdot E_c}{m}} = \sqrt{\frac{2 \cdot 1,92 \cdot 10^{-18}}{9,1 \cdot 10^{-31}}} = 4,8 \cdot 10^4 \frac{m}{s}$$

$$\lambda = \frac{h}{m \cdot v} = \frac{6,62 \cdot 10^{-34}}{9,1 \cdot 10^{-31} \cdot 4,8 \cdot 10^4} = \boxed{8,26 \cdot 10^{-12} m}$$

$$⑥ \quad M = 163$$

$$a) \quad n = \frac{m}{M} = \frac{40}{163} = \boxed{0,245 \text{ mol}}$$

$$b) \quad N = n \cdot N_A = 0,245 \cdot 6,022 \cdot 10^{23} = \boxed{1,48 \cdot 10^{23} \text{ moléculas}}$$

$$c) \quad N: 0,245 \cdot 2 = \boxed{0,49 \text{ mol}}; H: 0,245 \cdot 8 = \boxed{1,96 \text{ mol}}$$

$$Se: 0,245 \cdot 1 = \boxed{0,245 \text{ mol}}; O: 0,245 \cdot 3 = \boxed{0,735 \text{ mol}}$$

$$d) \quad N: 0,49 \cdot 14 = \boxed{6,86 g}; H: 1,96 \cdot 1 = \boxed{1,96 g}$$

$$Se: 0,245 \cdot 78,96 = \boxed{19,4 g}; O: 0,735 \cdot 16 = \boxed{11,8 g}$$

$$e) \quad N: 1,48 \cdot 10^{23} \cdot 2 = \boxed{2,96 \cdot 10^{23}}$$

$$H: 1,48 \cdot 10^{23} \cdot 8 = \boxed{1,18 \cdot 10^{24}}$$

$$Se: 1,48 \cdot 10^{23} \cdot 1 = \boxed{1,48 \cdot 10^{23}}; O: 1,48 \cdot 10^{23} \cdot 3 = \boxed{4,44 \cdot 10^{23}}$$

(175)

$$\textcircled{7} \text{ a) } n = \frac{M_{\text{molecular}}}{M_{\text{empírica}}} = \frac{176}{12 \cdot 4 + 8 \cdot 1 + 32} = \frac{176}{88} = 2$$

Fórmula molecular: $\boxed{\text{C}_8\text{H}_{16}\text{S}_2}$

$$\text{b) } N_{\text{H}} = 25 \text{ g } \cancel{\text{C}_8\text{H}_{16}\text{S}_2} \cdot \frac{1 \text{ mol compuesto}}{176 \text{ g } \cancel{\text{C}_8\text{H}_{16}\text{S}_2}}$$

$$\cdot \frac{16 \text{ mol H}}{1 \text{ mol compuesto}} \cdot \frac{6'022 \cdot 10^{23} \text{ átomos H}}{1 \text{ mol H}} = \boxed{1'37 \cdot 10^{24} \text{ átomos}}$$

$$\text{c) } P \cdot V = n \cdot R \cdot T \Rightarrow P \cdot V = \frac{m}{M} \cdot R \cdot T \Rightarrow$$

$$\Rightarrow P = \frac{m \cdot R \cdot T}{M \cdot V} = \frac{2 \cdot 0'082 \cdot (120 + 273)}{176 \cdot 1'5} = \boxed{0'244 \text{ atm}}$$

$\textcircled{8} \text{ a) }$

$$m = 40 \frac{\text{g}}{\text{mol}} \cdot \frac{1 \text{ mol}}{6'022 \cdot 10^{23} \text{ átomos}} = \boxed{6'64 \cdot 10^{-23} \text{ g}}$$

$$\text{b) } M = 11 + 3 \cdot 35'5 = 117'5$$

$$N = 5 \text{ g } \cancel{\text{BCl}_3} \cdot \frac{1 \text{ mol BCl}_3}{117'5 \text{ g } \cancel{\text{BCl}_3}} \cdot \frac{6'022 \cdot 10^{23} \text{ moléculas BCl}_3}{1 \text{ mol BCl}_3} = \boxed{2'56 \cdot 10^{22} \text{ moléculas}}$$

$$\text{c) } M = 111$$

$$N = 2'8 \text{ g } \cancel{\text{CaCl}_2} \cdot \frac{1 \text{ mol CaCl}_2}{111 \text{ g } \cancel{\text{CaCl}_2}} \cdot \frac{2 \cdot 6'022 \cdot 10^{23} \text{ iones Cl}^-}{1 \text{ mol CaCl}_2} =$$

$$= \boxed{3'04 \cdot 10^{22} \text{ iones}} \quad (176)$$

9) a) $M = 18 \frac{\text{g}}{\text{mol}}$
 $n = 25 \cancel{\text{g}} \cdot \frac{1 \cancel{\text{g}}}{1 \cancel{\text{mol}}} \cdot \frac{1 \text{ mol}}{18 \text{ g}} = \boxed{1.39 \text{ mol}}$

b)
 $N_H = 1.39 \text{ mol H}_2\text{O} \cdot \frac{2 \cdot 6.022 \cdot 10^{23} \text{ átomos H}}{1 \text{ mol H}_2\text{O}} =$
 $= \boxed{1.67 \cdot 10^{24} \text{ átomos H}}$

c)
 $m = 18 \frac{\text{g}}{\cancel{\text{mol}}} \cdot \frac{1 \cancel{\text{mol}}}{6.022 \cdot 10^{23} \text{ moléculas}} = \boxed{2.99 \cdot 10^{-23} \text{ g}}$

10) $M = 207 + 8 \cdot 12 + 20 \cdot 1 = 323 \frac{\text{g}}{\text{mol}}$

a)
 $N = 12.94 \text{ g} \cdot \frac{1 \cancel{\text{mol}}}{323 \text{ g}} \cdot \frac{6.022 \cdot 10^{23} \text{ moléculas}}{1 \cancel{\text{mol}}} = \boxed{2.41 \cdot 10^{22} \text{ moléculas}}$

b)
 $n = 1 \cancel{\text{g Pb}} \cdot \frac{1 \text{ mol Pb(C}_2\text{H}_5)_4}{207 \cancel{\text{g Pb}}} = \boxed{4.83 \cdot 10^{-3} \text{ mol}}$

c)
 $m = 207 \frac{\text{g}}{\cancel{\text{mol}}} \cdot \frac{1 \cancel{\text{mol}}}{6.022 \cdot 10^{23} \text{ átomos}} = \boxed{3.44 \cdot 10^{-22} \text{ g}}$

$$(11) \quad P \cdot V = n \cdot R \cdot T \Rightarrow P \cdot V = \frac{m}{M} \cdot R \cdot T \Rightarrow$$

$$\Rightarrow M = \frac{m \cdot R \cdot T}{P \cdot V} = \frac{30 \cdot 0.082 \cdot (273 + 40)}{2 \cdot 6.64} = 58 \frac{\text{g}}{\text{mol}}$$

$$x = \frac{M_{\text{molecular}}}{M_{\text{empírica}}} = \frac{58}{12 \cdot 2 + 5} = \frac{58}{29} = 2$$

Fórmula molecular: $\boxed{\text{C}_4\text{H}_{10}}$

$$(12) \quad M = 63.54 + 2 \cdot 14 + 6 \cdot 1 + 35.5 = 133$$

$$\text{Cu: } \frac{63.54 \cdot 100}{133} = \boxed{47.8\%}$$

$$\text{N: } \frac{2 \cdot 14 \cdot 100}{133} = \boxed{21.1\%}$$

$$\text{H: } \frac{3 \cdot 2 \cdot 100}{133} = \boxed{4.51\%}$$

$$\text{Cl: } \frac{35.5 \cdot 100}{133} = \boxed{26.7\%}$$

13) * Porcentaje de N: $100 - 30,3 - 4,2 - 53,8 = 11,7\%$

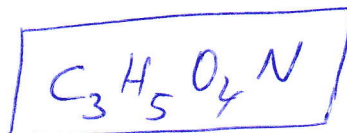
* Fórmula empírica:

$$C: \frac{30,3}{12} = 2,53 \rightarrow \frac{2,53}{0,836} \approx 3$$

$$H: \frac{4,2}{1} = 4,2 \rightarrow \frac{4,2}{0,836} \approx 5$$

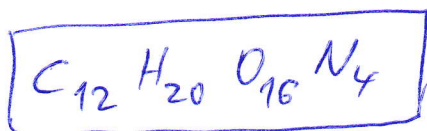
$$O: \frac{53,8}{16} = 3,36 \rightarrow \frac{3,36}{0,836} \approx 4$$

$$N: \frac{11,7}{14} = 0,836 \rightarrow \frac{0,836}{0,836} = 1$$



* Fórmula molecular:

$$x = \frac{M_{\text{molecular}}}{M_{\text{empírica}}} = \frac{476}{3 \cdot 12 + 5 + 4 \cdot 16 + 14} = \frac{476}{119} = 4$$



14) $M = 159,7$

$$N_{Fe} = 1 \text{ ton hematites} \cdot \frac{10^6 \text{ g hematites}}{1 \text{ ton hematites}}$$

$$\cdot \frac{80 \text{ g Fe}_2\text{O}_3}{100 \text{ g hematites}} \cdot \frac{1 \text{ mol Fe}_2\text{O}_3}{159,7 \text{ g Fe}_2\text{O}_3}$$

$$\cdot \frac{2 \text{ mol Fe}}{1 \text{ mol Fe}_2\text{O}_3} \cdot \frac{6,022 \cdot 10^{23} \text{ átomos Fe}}{1 \text{ mol Fe}} =$$

$$= \frac{10^6 \cdot 80 \cdot 2 \cdot 6,022 \cdot 10^{23}}{100 \cdot 159,7} = \boxed{6,03 \cdot 10^{23} \text{ átomos}}$$

(179)

$$(15) \quad m_s = 80 \text{ g} ; m_d = 187 \text{ g} ; m_D = 80 + 187 = 267 \text{ g} ;$$

$$V_D = \frac{m_D}{d_D} = \frac{267}{1.175} = 227 \text{ ml} ; V_d = 187 \text{ ml} ;$$

$$V_s = 227 - 187 = 40 \text{ ml} ; M_s = 98 ; M_d = 18 ;$$

$$n_s = \frac{m_s}{M_s} = \frac{80}{98} = 0.816 \text{ mol} ; n_d = \frac{m_d}{M_d} = \frac{187}{18} = 10.4 \text{ mol}$$

$$* \text{ Porcentaje en masa} = \frac{m_s \cdot 100}{m_D} = \frac{80 \cdot 100}{267} = \boxed{30\%}$$

$$* \text{ Porcentaje en volumen} = \frac{V_s \cdot 100}{V_D} = \frac{40 \cdot 100}{227} = \boxed{17.6\%}$$

* Masa por unidad de volumen:

$$c = \frac{m_s}{V_D} = \frac{80 \text{ g}}{0.227 \text{ l}} = \boxed{352 \frac{\text{g}}{\text{l}}}$$

$$* \text{ Molaridad: } c_M = \frac{n_s}{V_D} = \frac{0.816 \text{ mol}}{0.227 \text{ l}} = \boxed{3.6 \text{ M}}$$

$$* \text{ Molalidad: } c_m = \frac{n_s}{m_d} = \frac{0.816 \text{ mol}}{0.187 \text{ kg}} = \boxed{4.36 \text{ m}}$$

* Fracción molar:

$$x = \frac{n_s}{n_s + n_d} = \frac{0.816}{0.816 + 10.4} = \boxed{0.0728}$$

$$(16) \quad M = 98 \frac{\text{g}}{\text{mol}}$$

$$\begin{aligned} * \text{Porcentaje en masa} &= \frac{2 \cancel{\text{mol}}}{\cancel{1}} \cdot \frac{98 \text{ g}}{1 \cancel{\text{mol}}} \cdot \frac{1 \cancel{\text{L}} \cdot 100}{111 \cdot 10^3 \text{ g}} = \\ &= \boxed{17.7\%} \end{aligned}$$

$$1 \text{ l disolución} \rightarrow 1110 \text{ g disolución} \left\{ \begin{array}{l} 1110 \cdot 0.177 = 196.5 \text{ g } \text{H}_2\text{SO}_4 \\ 1110 - 196.5 = 913.5 \text{ g } \text{H}_2\text{O} \end{array} \right.$$

$$913.5 \text{ g H}_2\text{O} = 913.5 \text{ ml H}_2\text{O} = V_d$$

$$V_s = V_D - V_d = 100 - 913.5 = 86.5 \text{ ml}$$

$$* \text{Porcentaje en volumen} = \frac{V_s \cdot 100}{V_D} = \frac{86.5 \cdot 100}{100} = \boxed{86.5\%}$$

* Masa por unidad de volumen:

$$c = 2 \frac{\cancel{\text{mol}}}{\cancel{\text{l}}} \cdot \frac{98 \text{ g}}{1 \cancel{\text{mol}}} = \boxed{196 \frac{\text{g}}{\text{l}}}$$

* Molaridad:

$$c_m = \frac{n_s}{m_d} = \frac{\frac{196.5}{98}}{0.9135} = \frac{196.5}{98 \cdot 0.9135} = \boxed{2.2 \text{ m}}$$

* Fracción molar:

$$x = \frac{n_s}{n_s + n_d} = \frac{\frac{196.5}{98}}{\frac{196.5}{98} + \frac{913.5}{18}} = \boxed{0.038}$$

17) a) $c_{M1} \cdot V_1 = c_{M2} \cdot V_2$

$$V_1 = \frac{c_{M2} \cdot V_2}{c_{M1}} = \frac{1'5 \cdot 100}{10} = \boxed{15 \text{ ml}}$$

b) $M = 98 \frac{\text{g}}{\text{mol}}$

Porcentaje en masa = $10 \frac{\text{mol}}{\cancel{\text{L}}} \cdot \frac{98 \text{ g}}{1 \text{ mol}} \cdot \frac{1 \cancel{\text{ l disolución}}}{1000 \cancel{\text{ ml}}}$

$\cdot \frac{1 \cancel{\text{ ml disolución}}}{1'53 \text{ g disolución}} \cdot 100 = \boxed{64'1\%}$

Fracción molar = $\frac{n_s}{n_s + n_d} = \frac{\frac{64'1}{98}}{\frac{64'1}{98} + \frac{35'9}{18}} = \boxed{0'247}$

18) a) $c_{M1} \cdot V_1 = c_{M2} \cdot V_2$

$$V_1 = \frac{c_{M2} \cdot V_2}{c_{M1}} = \frac{0'75 \cdot 100}{12} = \boxed{6'25 \text{ ml}}$$

b)

Porcentaje en masa = $12 \frac{\text{mol}}{\cancel{\text{L}}} \cdot \frac{63 \text{ g}}{1 \text{ mol}} \cdot \frac{1 \cancel{\text{ l disolución}}}{1000 \cancel{\text{ ml}}}$

$\cdot \frac{1 \cancel{\text{ ml disolución}}}{1'3 \text{ g disolución}} \cdot 100 = \boxed{58'2\%}$

Fracción molar = $\frac{n_s}{n_s + n_d} = \frac{\frac{58'2}{63}}{\frac{58'2}{63} + \frac{41'8}{18}} = \boxed{0'285}$

(19) a)

$$c_{M1} = \frac{36 \text{ g HCl}}{100 \text{ g disolución}} \cdot \frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}} \cdot \frac{1.18 \text{ g disol.}}{1 \text{ ml disol.}}$$

$$\frac{1000 \text{ ml disol.}}{1 \text{ l disol.}} = \frac{36 \cdot 1.18 \cdot 1000}{100 \cdot 36.5} = \boxed{11.6 \text{ M}}$$

$$X = \frac{n_s}{n_s + n_d} = \frac{\frac{36}{36.5}}{\frac{36}{36.5} + \frac{64}{18}} = \boxed{0.217}$$

b) $c_{M1} \cdot V_1 = c_{M2} \cdot V_2$

$$V_1 = \frac{c_{M2} \cdot V_2}{c_{M1}} = \frac{2 \cdot 250}{11.6} = \boxed{43.1 \text{ ml}}$$

(20) $n_m?$

$V = 40 \text{ l O}_2$

cc.NN.

$P = 16$

$$M = 16 \cdot 2 = 32 \frac{\text{g}}{\text{mol}}$$

$$P \cdot V = n \cdot R \cdot T \Rightarrow P \cdot V = \frac{m}{M} \cdot R \cdot T$$

$$m = \frac{P \cdot V \cdot M}{R \cdot T} =$$

$$= \frac{1 \cdot 40 \cdot 32}{0.082 \cdot 273} = \boxed{57.2 \text{ g}}$$

(21) $T_1 = 30^\circ\text{C}$

$P_1 = 1.5 \text{ atm}$

$T_2 = ?$

$V_2 = \frac{V_1}{2}$

$P_2 = 5 \text{ atm}$

$$\frac{P_1 \cdot V_1}{T_1} = \frac{P_2 \cdot V_2}{T_2}$$

$$T_2 = \frac{P_2 \cdot V_2 \cdot T_1}{P_1 \cdot V_1} =$$

$$= \frac{5 \cdot \frac{V_1}{2} \cdot 303}{1.5 \cdot V_1} = \frac{5 \cdot 303}{2 \cdot 1.5} = \boxed{505 \text{ K}}$$

(22) $d = ?$

$M = 108$

$T = -10^\circ\text{C} = 263 \text{ K}$

$P = 740 \text{ mm Hg} = \frac{740}{760} =$

$= 0.974 \text{ atm}$

$$d = \frac{P \cdot M}{R \cdot T} =$$

$$= \frac{0.974 \cdot 108}{0.082 \cdot 263} = \boxed{4.88 \frac{\text{g}}{\text{l}}}$$

(23) $d = ?$

CC.NN.

$\text{CO}_2 : 30\%$

$\text{SO}_2 : 20\%$

$\text{O}_2 : 35\%$

$\text{N}_2 : 15\%$

$$\bar{M} = \sum x_i \cdot M_i =$$

$$= 0.3 \cdot 44 + 0.2 \cdot 64 + 0.35 \cdot 32 + 0.15 \cdot 28 =$$

$$= 41.4 \frac{\text{g}}{\text{mol}}$$

$$d = \frac{P \cdot M}{R \cdot T} = \frac{1 \cdot 41.4}{0.082 \cdot 273} = \boxed{1.85 \frac{\text{g}}{\text{l}}}$$

24) $\hat{P}_2?$

$$T_1 = T_2$$

$$V_1 = 10 \text{ l}$$

$$V_2 = 7 \text{ l}$$

$$P_1 = 1\frac{1}{2} \text{ atm}$$

$$\frac{P_1 \cdot V_1}{\cancel{T_1}} = \frac{P_2 \cdot V_2}{\cancel{T_2}} \Rightarrow$$

$$\Rightarrow P_2 = \frac{P_1 \cdot V_1}{V_2} = \frac{1\frac{1}{2} \cdot 10}{7} = \boxed{1\frac{1}{7} \text{ atm}}$$

27) a) $a \text{ C}_8\text{H}_{18} + b \text{ O}_2 \rightarrow c \text{ CO}_2 + d \text{ H}_2\text{O}$

$$\text{C: } 8 \cdot a = c$$

$$\text{H: } 18 \cdot a = 2 \cdot d$$

$$\text{O: } 2 \cdot b = 2 \cdot c + d$$

$$a = 1$$

$$c = 8$$

$$d = \frac{18 \cdot a}{2} = \frac{18 \cdot 1}{2} = 9$$

$$a = 1 \rightarrow 2$$

$$b = \frac{25}{2} \rightarrow 25$$

$$c = 8 \rightarrow 16$$

$$d = 9 \rightarrow 18$$

$$2 \cdot b = 16 + 9 = 25$$

b) $a \text{ H}_2\text{SO}_4 + b \text{ S} \rightarrow c \text{ SO}_2 + d \text{ H}_2\text{O}$

$$\text{H: } 2 \cdot a = 2 \cdot d$$

$$\text{S: } a + b = c$$

$$\text{O: } 4a = 2c + d$$

$$4 \cdot 1 = 2 \cdot c + 1$$

$$3 = 2 \cdot c$$

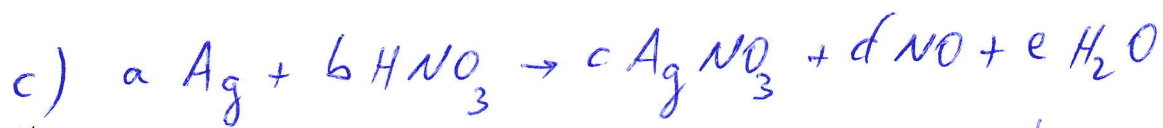
$$a = 1 \rightarrow 2$$

$$b = \frac{1}{2} \rightarrow 1$$

$$c = \frac{3}{2} \rightarrow 3$$

$$d = 1 \rightarrow 2$$

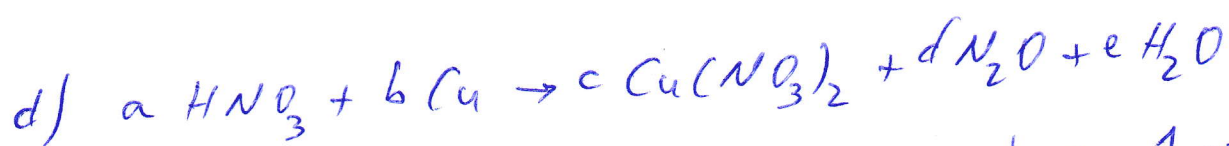
$$1 + b = \frac{3}{2} ; b = \frac{3}{2} - 1 = \frac{1}{2}$$



$$\left. \begin{array}{l} \text{Ag: } a = c \\ \text{H: } b = 2e \\ \text{N: } b = c + d \\ \text{O: } 3b = 3c + d + e \end{array} \right\} \left. \begin{array}{l} 2e = c + d \\ 3 \cdot 2 \cdot e = 3c + d + e \\ 2e = c + d \\ 5e = 3c + d \end{array} \right\} \left. \begin{array}{l} a = 1 \rightarrow 3 \\ b = \frac{4}{3} \rightarrow 4 \\ c = 1 \rightarrow 3 \\ d = \frac{1}{3} \rightarrow 1 \\ e = \frac{2}{3} \rightarrow 2 \end{array} \right.$$

$$3e = 2c \Rightarrow e = \frac{2 \cdot c}{3} = \frac{2 \cdot 1}{3} = \frac{2}{3}$$

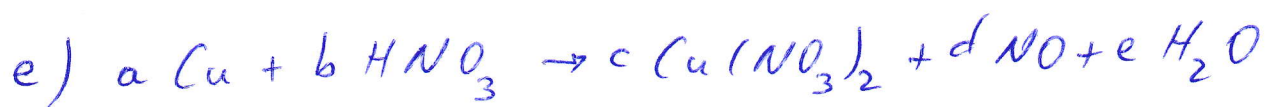
$$b = 2 \cdot e = 2 \cdot \frac{2}{3} = \frac{4}{3} ; d = 2 \cdot e - c = \frac{4}{3} - 1 = \frac{1}{3}$$



$$\left. \begin{array}{l} \text{H: } a = 2e \\ \text{N: } a = 2c + 2d \\ \text{O: } 3a = 6c + d + e \\ \text{Cu: } b = c \end{array} \right\} \left. \begin{array}{l} 1 = 2c + 2d \\ 3 = 6c + d + \frac{1}{2} \\ 1 = 2c + 2d \\ \frac{5}{2} = 6c + d \end{array} \right\} \left. \begin{array}{l} a = 1 \rightarrow 10 \\ b = \frac{2}{5} \rightarrow 4 \\ c = \frac{2}{5} \rightarrow 4 \\ d = \frac{1}{10} \rightarrow 1 \\ e = \frac{1}{2} \rightarrow 5 \end{array} \right.$$

$$\left. \begin{array}{l} 1 = 2c + 2d \\ -5 = -12c - 2d \end{array} \right\} \left. \begin{array}{l} d = \frac{5}{2} - 6c = \frac{5}{2} - \frac{12}{5} = \\ = \frac{25 - 24}{10} = \frac{1}{10} \end{array} \right.$$

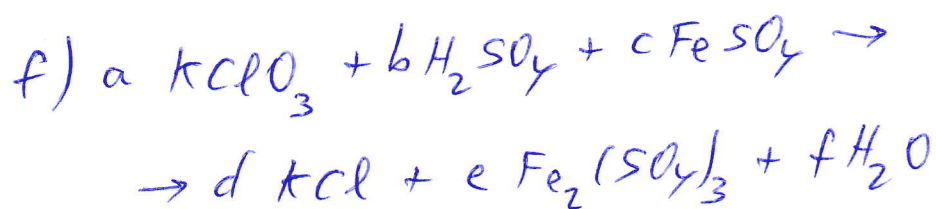
$$\begin{array}{l} -4 = -10 \cdot c \\ c = \frac{4}{10} = \frac{2}{5} \end{array}$$



$$\left. \begin{array}{l} \text{Cu: } a = c \\ \text{H: } b = 2e \\ \text{N: } b = 2c + d \\ \text{O: } 3b = 6c + d + e \end{array} \right\} \begin{array}{l} 2e = 2c + d \\ 3 \cdot 2e = 6c + d + e \\ 2e = 2c + d \\ 5e = 6c + d \end{array} \left| \begin{array}{l} a = 1 \rightarrow 3 \\ b = \frac{8}{3} \rightarrow 8 \\ c = 1 \rightarrow 3 \\ d = \frac{2}{3} \rightarrow 2 \\ e = \frac{4}{3} \rightarrow 4 \end{array} \right.$$

$$3e = 4c \Rightarrow e = \frac{4 \cdot c}{3} = \frac{4 \cdot 1}{3} = \frac{4}{3}$$

$$b = 2 \cdot e = 2 \cdot \frac{4}{3} = \frac{8}{3} ; d = 2e - 2c = \frac{8}{3} - 2 = \frac{2}{3}$$



$$\left. \begin{array}{l} \text{K: } a = d \\ \text{Cl: } a = d \\ \text{O: } 3a + 4b + 4c = 12e + f \\ \text{H: } 2b = 2f \\ \text{S: } b + c = 3e \\ \text{Fe: } c = 2e \end{array} \right\} \begin{array}{l} a = 1 \\ b = 3 \\ c = 6 \\ d = 1 \\ e = 3 \\ f = 3 \end{array}$$

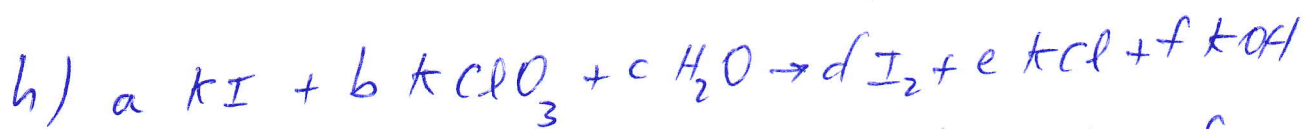
$$3 + 4b + 4c = 12e + f ; b = f$$

$$b + 2e = 3e \Rightarrow b = e ; c = 3e - b = 3b - b = 2b$$

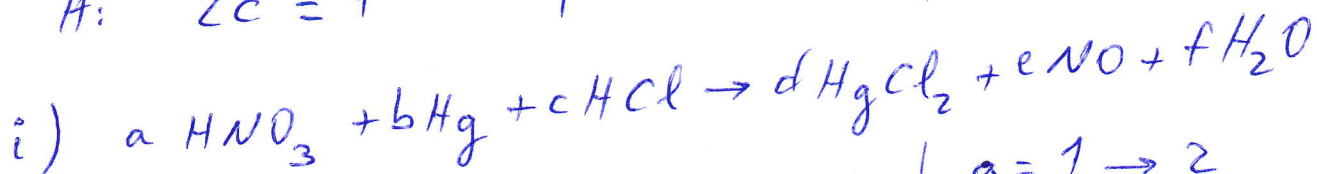
$$3 + 4b + 4 \cdot 2b = 12 \cdot b + b ; 3 + 12 \cdot b = 13 \cdot b \Rightarrow b = 3$$



$$\begin{array}{l|l|l} \text{H: } a + 2b = 2e & 1 + 2b = 4 & a = 1 \rightarrow 2 \\ \text{N: } a = c & & b = \frac{3}{2} \rightarrow 3 \\ \text{O: } 3a = c + e & & c = 1 \rightarrow 2 \\ \text{S: } b = d & & d = \frac{3}{2} \rightarrow 3 \\ & & e = 2 \rightarrow 4 \end{array}$$



$$\begin{array}{l|l|l} \text{K: } a + b = e + f & a + b = e + f & a = 1 \rightarrow 6 \\ \text{I: } a = 2d & a = f = 1 & b = \frac{1}{6} \rightarrow 1 \\ \text{Cl: } b = e & c = \frac{f}{2} = \frac{1}{2} & c = \frac{1}{2} \rightarrow 3 \\ \text{O: } 3b + c = f & 3b + \frac{1}{2} = 1 & d = \frac{1}{2} \rightarrow 3 \\ \text{H: } 2c = f & & e = \frac{1}{6} \rightarrow 1 \\ & & f = 1 \rightarrow 6 \end{array}$$



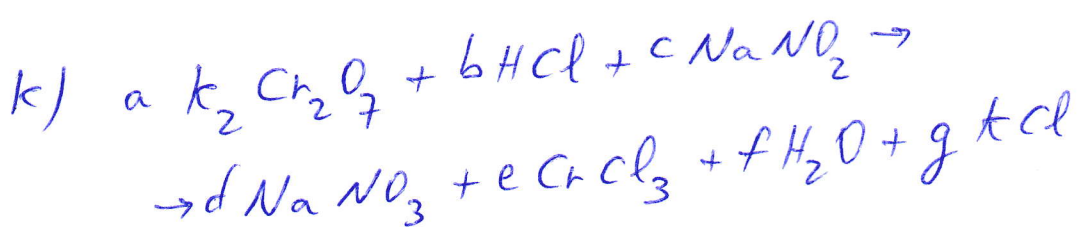
$$\begin{array}{l|l|l} \text{H: } a + c = 2f & 1 + c = 4 & a = 1 \rightarrow 2 \\ \text{N: } a = e & d = \frac{c}{2} = \frac{3}{2} & b = \frac{3}{2} \rightarrow 3 \\ \text{O: } 3a = e + f & & c = 3 \rightarrow 6 \\ \text{Hg: } b = d & & d = \frac{3}{2} \rightarrow 3 \\ \text{Cl: } c = 2d & & e = 1 \rightarrow 2 \\ & & f = 2 \rightarrow 4 \end{array}$$



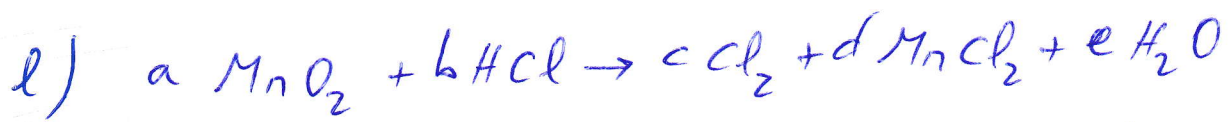
$$\begin{array}{l|l|l} \text{Ag: } 2a = c & 2f = c + d & a = 1 \rightarrow 3 \\ \text{S: } a = e & 3 \cdot 2f = 3c + d + f & b = \frac{8}{3} \rightarrow 8 \\ \text{H: } b = 2f & 2f = c + d & c = 2 \rightarrow 6 \\ \text{N: } b = c + d & 5f = 3c + d & d = \frac{2}{3} \rightarrow 2 \\ \text{O: } 3b = 3c + d + f & 3f = 2c & e = 1 \rightarrow 3 \\ & & f = \frac{4}{3} \rightarrow 4 \end{array}$$

$$f = \frac{2 \cdot c}{3} = \frac{2 \cdot 2}{3} = \frac{4}{3} ; b = 2 \cdot f = 2 \cdot \frac{4}{3} = \frac{8}{3}$$

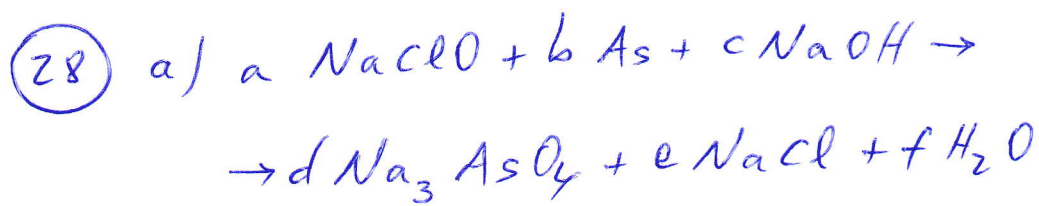
$$d = b - c = \frac{8}{3} - 2 = \frac{2}{3}$$



$$\begin{array}{l|l|l} \text{K: } 2a = g & b = 3 \cdot 2 + 2 = 8 & a = 1 \\ \text{Cr: } 2a = e & f = \frac{b}{2} = \frac{8}{2} = 4 & b = 8 \\ \text{O: } 7a + 2c = 3d + f & 7 + 2d = 3d + 4 & c = 3 \\ \text{H: } b = 2f & 3 = d & d = 3 \\ \text{Cl: } b = 3e + g & & e = 2 \\ \text{Na: } c = d & & f = 4 \\ \text{N: } c = d & & g = 2 \end{array}$$



$$\left. \begin{array}{l} \text{Mn: } a = d \\ \text{O: } 2a = e \\ \text{H: } b = 2e \\ \text{Cl: } b = 2c + 2d \end{array} \right\} \begin{array}{l} 4 = 2c + 2 \\ a = 1 \\ b = 4 \\ c = 1 \\ d = 1 \\ e = 2 \end{array}$$



$$\left. \begin{array}{l} \text{Na: } a + c = 3d + e \\ \text{Cl: } a = e \\ \text{O: } a + c = 4d + f \\ \text{As: } b = d \\ \text{H: } c = 2f \end{array} \right\} \begin{array}{l} a = 1 \rightarrow 5 \\ b = \frac{2}{5} \rightarrow 2 \\ c = \frac{6}{5} \rightarrow 6 \\ d = \frac{2}{5} \rightarrow 2 \\ e = 1 \rightarrow 5 \\ f = \frac{3}{5} \rightarrow 3 \end{array}$$

$$a + c = 3d + e \Rightarrow c = 3d$$

$$\left. \begin{array}{l} 1 + c = 4d + f \\ c = 2f \end{array} \right\} \begin{array}{l} d = \frac{c}{3} \\ f = \frac{c}{2} \end{array}$$

$$1 + c = 4 \cdot \frac{c}{3} + \frac{c}{2}$$

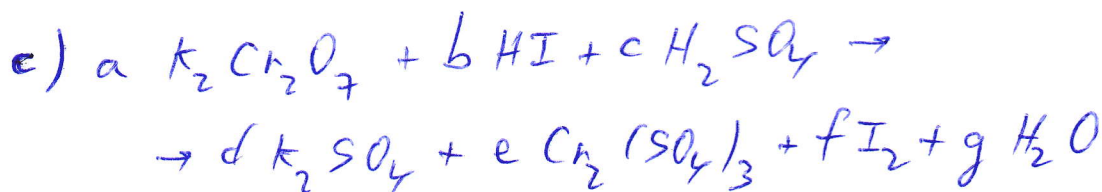
$$6 + 6c = 8c + 3c \Rightarrow 6 = 5c \Rightarrow c = \frac{6}{5}$$



$$\left. \begin{array}{l} \text{Ca: } a = e \\ \text{S: } a = d \\ \text{H: } b = 2f \\ \text{N: } b = c + 2e \\ \text{O: } 3b = c + 2d + 6e + f \end{array} \right\} \begin{array}{l} a = 1 \\ b = 4 \\ c = 2 \\ d = 1 \\ e = 1 \\ f = 2 \end{array}$$

$$\left. \begin{array}{l} b = c + 2 \\ 3b = c + 2 + 6 + \frac{b}{2} \end{array} \right\} \begin{array}{l} b = c + 2 \\ \frac{5b}{2} = c + 8 \end{array} \left\} \begin{array}{l} \frac{5b}{2} - b = 6 \\ \frac{3b}{2} = 6 \end{array} \right.$$

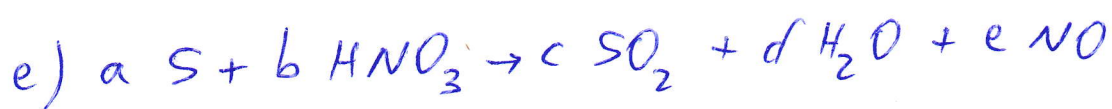
$$\frac{3b}{2} = 6 \Rightarrow b = \frac{6 \cdot 2}{3} = 4 ; c = b - 2 = 4 - 2 = 2$$



$$\left. \begin{array}{l} \text{K: } 2a = 2d \\ \text{Cr: } 2a = 2e \\ \text{O: } 7a + 4c = 4d + 12e + g \\ \text{H: } b + 2c = 2g \\ \text{I: } b = 2f \\ \text{S: } c = d + 3e \end{array} \right\} \begin{array}{l} c = 1 + 3 = 4 \\ 7 + 16 = 4 + 12 + g \\ b + 8 = 14 \\ a = 1 \\ b = 6 \\ c = 4 \\ d = 1 \\ e = 1 \\ f = 3 \\ g = 7 \end{array}$$



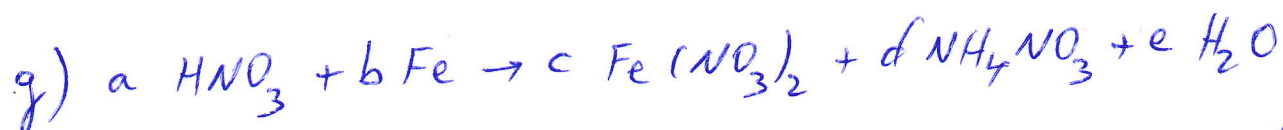
$$\begin{array}{l|l|l} \text{H:} & a + b = 2d & 1 + b = 6 \\ \text{I:} & a + b = 2c & 1 + 5 = 2 \cdot c \\ \text{O:} & 3a = d & d = 3 \end{array} \quad \begin{array}{l} a = 1 \\ b = 5 \\ c = 3 \\ d = 3 \end{array}$$



$$\begin{array}{l|l|l} \text{S:} & a = c & a = 1 \rightarrow 3 \\ \text{H:} & b = 2d & b = \frac{4}{3} \rightarrow 4 \\ \text{N:} & b = e & c = 1 \rightarrow 3 \\ \text{O:} & 3b = 2c + d + e & d = \frac{2}{3} \rightarrow 2 \\ & & e = \frac{4}{3} \rightarrow 4 \end{array} \quad \begin{array}{l} 3b = 2 + \frac{b}{2} + b \\ 2b - \frac{b}{2} = 2 \\ \frac{3}{2} \cdot b = 2 \end{array}$$



$$\begin{array}{l|l|l} \text{H:} & a = 2e & a = 1 \rightarrow 8 \\ \text{N:} & a = d & b = \frac{1}{8} \rightarrow 1 \\ \text{O:} & 3a = 4c + 2d + e & c = \frac{1}{8} \rightarrow 1 \\ \text{Pb:} & b = c & d = 1 \rightarrow 8 \\ \text{S:} & b = c & e = \frac{1}{2} \rightarrow 4 \end{array} \quad \begin{array}{l} 3 = 4c + 2 + \frac{1}{2} \\ \frac{1}{2} = 4c \end{array}$$



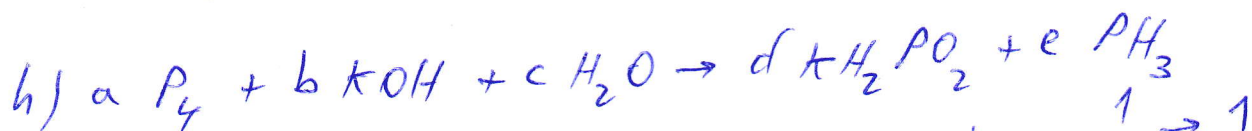
$$\begin{array}{lcl} \text{H:} & a = 4d + 2e & \\ \text{N:} & a = 2c + 2d & \\ \text{O:} & 3a = 6c + 3d + e & \\ \text{Fe:} & b = c & \end{array} \quad \left| \begin{array}{l} 1 = 4d + 2e \\ 1 = 2c + 2d \\ 3 = 6c + 3d + e \\ b = c \end{array} \right| \quad \left\{ \begin{array}{l} a = 1 \rightarrow 10 \\ b = \frac{2}{5} \rightarrow 4 \\ c = \frac{2}{5} \rightarrow 4 \\ d = \frac{1}{10} \rightarrow 1 \\ e = \frac{3}{10} \rightarrow 3 \end{array} \right.$$

$$e = \frac{1-4d}{2}$$

$$\left. \begin{array}{l} 1 = 2c + 2d \\ 3 = 6c + 3d + \frac{1-4d}{2} \end{array} \right\} \left. \begin{array}{l} 1 = 2c + 2d \\ 6 = 12c + 6d + 1 - 4d \end{array} \right\}$$

$$\left. \begin{array}{l} 1 = 2c + 2d \\ 5 = 12c + 2d \end{array} \right\} \quad 4 = 10 \cdot c \Rightarrow c = \frac{4}{10} = \frac{2}{5}$$

$$d = \frac{1-2c}{2} = \frac{1-\frac{4}{5}}{2} = \frac{1}{10}; \quad e = \frac{1-4d}{2} = \frac{1-\frac{4}{10}}{2} = \frac{6}{20} = \frac{3}{10}$$



$$\begin{array}{lcl} \text{P:} & 4a = d + e & \\ \text{K:} & b = d & \\ \text{O:} & b + c = 2d & \\ \text{H:} & b + 2c = 2d + 3e & \end{array} \quad \left| \begin{array}{l} 1 + \cancel{4} = \cancel{4} + 3e \\ 4a = 1 + \frac{1}{3} \\ 4a = \frac{4}{3} \end{array} \right| \quad \left\{ \begin{array}{l} a = \frac{1}{3} \rightarrow 1 \\ b = 1 \rightarrow 3 \\ c = 1 \rightarrow 3 \\ d = 1 \rightarrow 3 \\ e = \frac{1}{3} \rightarrow 1 \end{array} \right.$$



$$\begin{array}{l} \text{H: } a + 2b = 2d + 2e \\ \text{N: } a = c \\ \text{O: } 3a = c + 4d + e \\ \text{S: } b = d \end{array} \left| \begin{array}{l} 1 + 2b = 2d + 2e \\ 3 = 1 + 4d + e \\ 1 + \cancel{2b} = \cancel{2b} + 2e \\ 2 = 4d + e \end{array} \right. \left| \begin{array}{l} a = 1 \rightarrow 8 \\ b = \frac{3}{8} \rightarrow 3 \\ c = 1 \rightarrow 8 \\ d = \frac{3}{8} \rightarrow 3 \\ e = \frac{1}{2} \rightarrow 4 \end{array} \right.$$

$$2 - \frac{1}{2} = 4d ; \frac{3}{2} = 4d$$

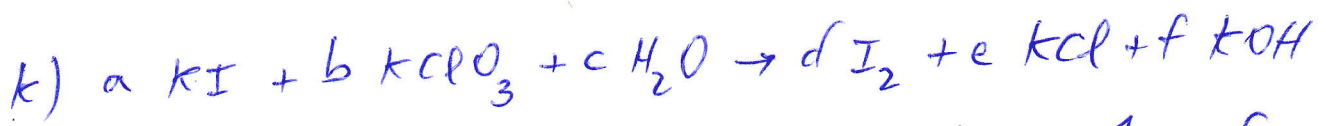


$$\begin{array}{l} \text{Co: } a = 2d \\ \text{Cl: } 2a + b = e \\ \text{K: } b + c = e \\ \text{O: } 3b + c = 3d + f \\ \text{H: } c = 2f \end{array} \left| \begin{array}{l} 2 + b = e \\ b + c = e \\ 3b + c = \frac{3}{2} + f \\ c = 2f \end{array} \right. \left| \begin{array}{l} a = 1 \rightarrow 6 \\ b = \frac{1}{6} \rightarrow 1 \\ c = 2 \rightarrow 12 \\ d = \frac{1}{2} \rightarrow 3 \\ e = \frac{13}{6} \rightarrow 13 \\ f = 1 \rightarrow 6 \end{array} \right.$$

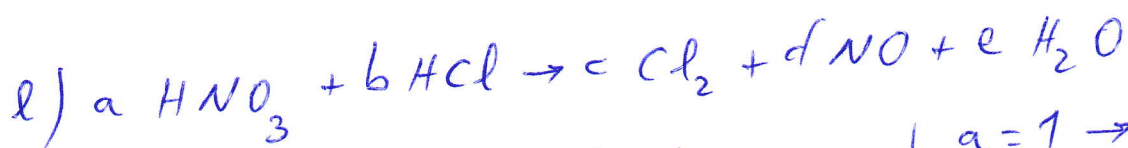
$$2 + \cancel{b} = \cancel{b} + c ; 3b + 2 = \frac{3}{2} + 1$$

$$3b = \frac{3}{2} + 1 - 2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$e = 2 + b = 2 + \frac{1}{6} = \frac{13}{6}$$



$$\begin{array}{lcl} \text{K:} & a + b = e + f & 1 + b = e + f \\ \text{I:} & a = 2d & \\ \text{Cl:} & b = e & \\ \text{O:} & 3b + c = f & \\ \text{H:} & 2c = f & \end{array} \quad \left| \begin{array}{l} 1 + b = e + f \\ 3b + \frac{1}{2} = 1 \\ 3b = \frac{1}{2} \end{array} \right| \quad \left| \begin{array}{l} a = 1 \rightarrow 6 \\ b = \frac{1}{6} \rightarrow 1 \\ c = \frac{1}{2} \rightarrow 3 \\ d = \frac{1}{2} \rightarrow 3 \\ e = \frac{1}{6} \rightarrow 1 \\ f = 1 \rightarrow 6 \end{array} \right.$$



$$\begin{array}{lcl} \text{H:} & a + b = 2e & 1 + b = 4 \\ \text{N:} & a = d & \\ \text{O:} & 3a = d + e & \\ \text{Cl:} & b = 2c & \end{array} \quad \left| \begin{array}{l} a = 1 \rightarrow 2 \\ b = 3 \rightarrow 6 \\ c = \frac{3}{2} \rightarrow 3 \\ d = 1 \rightarrow 2 \\ e = 2 \rightarrow 4 \end{array} \right.$$

$$(29) \quad m_{\text{HNO}_3} + m_{\text{H}_2\text{S}} = m_{\text{NO}} + m_{\text{H}_2\text{SO}_4} + m_{\text{H}_2\text{O}}$$

$$504 + a = 240 + 294 + 72 \Rightarrow a = \boxed{102 \text{ g}}$$

$$b = \frac{504 \cdot 500}{294} = \boxed{857 \text{ g}}$$

$$c = \frac{102 \cdot 500}{294} = \boxed{173 \text{ g}}; \quad d = \frac{240 \cdot 500}{294} = \boxed{408 \text{ g}}$$

$$b + c = d + 500 + e \Rightarrow 857 + 173 = 408 + 500 + e;$$

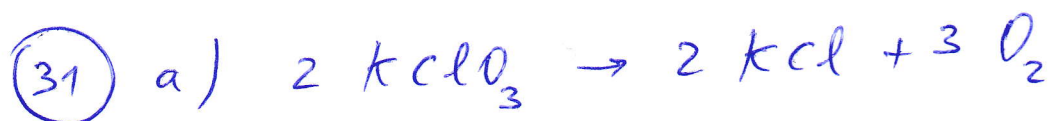
$$e = \boxed{122 \text{ g}}$$

$$(30) \quad a = \frac{40.5}{2} = \boxed{100 \text{ l}} ; \quad b = \frac{40.3}{2} = \boxed{60 \text{ l}}$$

$$c \rightarrow \text{es sólido} ; \quad d = \frac{40.8}{2} = \boxed{160 \text{ l}}$$

$$e = \frac{40.2}{60} = \boxed{133 \text{ cm}^3} ; \quad f = \frac{a \cdot 2}{b} = \frac{100 \cdot 2}{60} = \boxed{333 \text{ cm}^3}$$

$$g \rightarrow \text{es sólido} ; \quad h = \frac{2 \cdot d}{b} = \frac{2 \cdot 160}{60} = \boxed{533 \text{ cm}^3}$$



$$M_{\text{KClO}_3} = 122.5$$

$$* \text{ Moles de O}_2 : \quad n = \frac{P \cdot V}{R \cdot T} = \frac{2 \cdot 5}{0.082 \cdot 293} = 0.416 \text{ mol O}_2$$

* Masa de KClO_3 :

$$m = 0.416 \text{ mol O}_2 \cdot \frac{2 \text{ mol KClO}_3}{3 \text{ mol O}_2} \cdot \frac{122.5 \text{ g KClO}_3}{1 \text{ mol KClO}_3} = \boxed{34 \text{ g}}$$

$$b) \quad M_{\text{KCl}} = 74.5$$

* Masa de KCl :

$$m = 60 \text{ g sustancia} \cdot \frac{83 \text{ g KClO}_3}{100 \text{ g sustancia}} \cdot \frac{1 \text{ mol KClO}_3}{122.5 \text{ g KClO}_3} \cdot \frac{2 \text{ mol KCl}}{2 \text{ mol KClO}_3} \cdot \frac{70}{100} \cdot \frac{74.5 \text{ g KCl}}{1 \text{ mol KCl}} = \frac{60 \cdot 83 \cdot 70 \cdot 74.5}{100 \cdot 122.5 \cdot 100} = \boxed{212 \text{ g KCl}}$$



$$M_{\text{CaCO}_3} = 100$$

$$M_{\text{CaCl}_2} = 111$$

a) * Masa de caliza:

$$m = 2500 \text{ g } \cancel{\text{CaCl}_2} \cdot \frac{1 \text{ mol } \cancel{\text{CaCl}_2}}{111 \text{ g } \cancel{\text{CaCl}_2}} \cdot \frac{1 \text{ mol } \cancel{\text{CaCO}_3}}{1 \text{ mol } \cancel{\text{CaCl}_2}} \cdot \frac{100 \text{ g } \cancel{\text{CaCO}_3}}{1 \text{ mol } \cancel{\text{CaCO}_3}} \cdot \frac{100 \text{ g } \cancel{\text{caliza}}}{92 \text{ g } \cancel{\text{CaCO}_3}} \cdot \frac{1 \text{ kg } \cancel{\text{caliza}}}{1000 \text{ g } \cancel{\text{caliza}}} =$$

$$= \frac{2500 \cdot 100 \cdot 100}{111 \cdot 92 \cdot 1000} = \boxed{2'45 \text{ kg caliza}}$$

b) * Moles de CO_2 :

$$n = 2500 \text{ g } \cancel{\text{CaCl}_2} \cdot \frac{1 \text{ mol } \cancel{\text{CaCl}_2}}{111 \text{ g } \cancel{\text{CaCl}_2}} \cdot \frac{1 \text{ mol } \text{CO}_2}{1 \text{ mol } \cancel{\text{CaCl}_2}} =$$

$$= 22'5 \text{ mol } \text{CO}_2$$

* Volumen de CO_2 :

$$V = \frac{n \cdot R \cdot T}{P} = \frac{22'5 \cdot 0'082 \cdot 298}{1/2} = \boxed{458 \text{ l } \text{CO}_2}$$

33) a) $M_{\text{NaHCO}_3} = 84$

$$n = \frac{m}{M} = \frac{50}{84} = 0.595 \text{ mol NaHCO}_3$$

* Moles de CO_2 :

$$n = 0.595 \text{ mol } \cancel{\text{NaHCO}_3} \cdot \frac{1 \text{ mol CO}_2}{2 \text{ mol } \cancel{\text{NaHCO}_3}} = 0.298 \text{ mol CO}_2$$

* Volumen de CO_2 :

$$V = \frac{n \cdot R \cdot T}{P} = \frac{0.298 \cdot 0.082 \cdot 298}{1/2} = \boxed{6.07 \text{ l CO}_2}$$

b) $M_{\text{Na}_2\text{CO}_3} = 106$

* Masa de Na_2CO_3 :

$$m = 0.595 \text{ mol } \cancel{\text{NaHCO}_3} \cdot \frac{1 \text{ mol } \cancel{\text{Na}_2\text{CO}_3}}{2 \text{ mol } \cancel{\text{NaHCO}_3}} \cdot$$

$$\cdot \frac{106 \text{ g Na}_2\text{CO}_3}{1 \text{ mol } \cancel{\text{Na}_2\text{CO}_3}} \cdot \frac{83}{100} = \frac{0.595 \cdot 106 \cdot 83}{2 \cdot 100} =$$

$$= \boxed{26.2 \text{ g Na}_2\text{CO}_3}$$



$$M_{\text{KClO}_3} = 122.5$$

$$M_{\text{KCl}} = 74.5$$

* Masa de KClO_3 : $m = 5.095 = 4.75 \text{ g KClO}_3$

$$n = \frac{m}{M} = \frac{4.75}{122.5} = 0.0388 \text{ mol KClO}_3$$

a) * Masa de KCl :

$$m = 0.0388 \text{ mol KClO}_3 \cdot \frac{2 \text{ mol KCl}}{2 \text{ mol KClO}_3} \cdot \frac{74.5 \text{ g KCl}}{1 \text{ mol KCl}} \cdot \frac{83}{100} = \frac{0.0388 \cdot 74.5 \cdot 83}{100} = \boxed{2.4 \text{ g KCl}}$$

b) * Moles de O_2 :

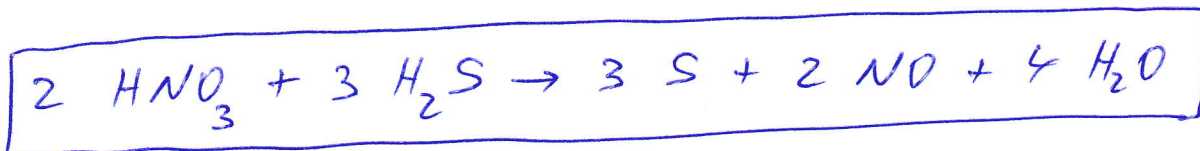
$$n = 0.0388 \text{ mol KClO}_3 \cdot \frac{3 \text{ mol O}_2}{2 \text{ mol KClO}_3} \cdot \frac{83}{100} = 0.0483 \text{ mol O}_2$$

* Volumen de O_2 :

$$V = \frac{n \cdot R \cdot T}{P} = \frac{0.0483 \cdot 0.082 \cdot 293}{\frac{720}{760}} = \boxed{1.23 \text{ l O}_2}$$



$$\begin{array}{l|l|l} \text{H:} & a + 2b = 2e & 1 + 2b = 4 \\ \text{N:} & a = d & a = 1 \rightarrow 2 \\ \text{O:} & 3a = d + e & b = \frac{3}{2} \rightarrow 3 \\ \text{S:} & b = c & c = \frac{3}{2} \rightarrow 3 \\ & & d = 1 \rightarrow 2 \\ & & e = 2 \rightarrow 4 \end{array}$$

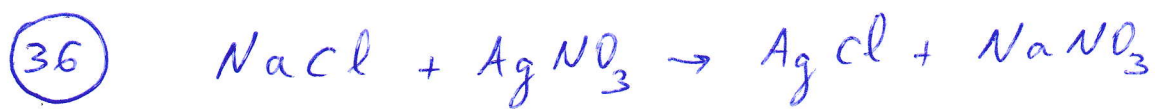


* Moles de H_2S :

$$\begin{aligned} n &= 0'2 \frac{\cancel{\text{mol HNO}_3}}{\cancel{\text{mol HNO}_3}} \cdot 0'5 \cancel{\text{L}} \cdot \frac{3 \text{ mol H}_2\text{S}}{2 \cancel{\text{mol HNO}_3}} = \\ &= \frac{0'2 \cdot 0'5 \cdot 3}{2} = 0'15 \text{ mol H}_2\text{S} \end{aligned}$$

* Volumen de H_2S :

$$V = \frac{n \cdot R \cdot T}{P} = \frac{0'15 \cdot 0'082 \cdot (273 + 60)}{1} = \boxed{4'10 \text{ L H}_2\text{S}}$$



a) $M_{\text{AgCl}} = 143.5$

* Moles de AgNO_3 : $n = c \cdot V = 0.5 \frac{\text{mol}}{\text{L}} \cdot 0.1 \text{ L} = 0.05 \text{ mol}$

* Moles de NaCl : $n = c \cdot V = 0.4 \frac{\text{mol}}{\text{L}} \cdot 0.1 \text{ L} = 0.04 \text{ mol}$

* Limitante: NaCl

* Masa de AgCl :

$$m = 0.04 \text{ mol } \cancel{\text{NaCl}} \cdot \frac{1 \text{ mol } \cancel{\text{AgCl}}}{1 \text{ mol } \cancel{\text{NaCl}}} \cdot \frac{143.5 \text{ g } \text{AgCl}}{1 \text{ mol } \cancel{\text{AgCl}}} =$$

$$= 0.04 \cdot 143.5 = \boxed{5.74 \text{ g } \text{AgCl}}$$

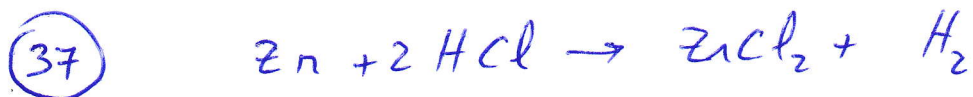
b) $M_{\text{AgNO}_3} = 170$

* Moles de reactivo en exceso:

$$n = 0.05 - 0.04 = 0.01 \text{ mol}$$

* Masa de reactivo en exceso:

$$m = n \cdot M = 0.01 \cdot 170 = \boxed{1.7 \text{ g}}$$



a) * Molaridad de la disolución de HCl:

$$C_M = \frac{37 \text{ g HCl}}{100 \text{ g disolución}} \cdot \frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}} \cdot \frac{1.18 \text{ g disolución}}{1 \text{ ml disolución}} \cdot \frac{1000 \text{ ml disolución}}{1 \text{ l disolución}} = \boxed{12 \text{ M}}$$

b) * Masa de Zn en la muestra:

$$m = 0.126 \text{ l ácido} \cdot 12 \frac{\text{mol HCl}}{\text{l}} \cdot \frac{1 \text{ mol Zn}}{2 \text{ mol HCl}} \cdot \frac{65.4 \text{ g Zn}}{1 \text{ mol Zn}} = \frac{0.126 \cdot 12 \cdot 65.4}{2} = 49.4 \text{ g Zn}$$

* Porcentaje de zinc en la muestra:

$$\text{Porcentaje} = \frac{m_{\text{Zn}} \cdot 100}{m_T} = \frac{49.4 \cdot 100}{50} = \boxed{98.8 \%}$$

(38) a) * Moles de zinc: $n = \frac{m}{M} = \frac{20}{65.4} = 0.306 \text{ mol}$

* Moles de HCl: $n = c \cdot V = 6 \frac{\text{mol}}{\text{L}} \cdot 0.2 \text{ L} = 1.2 \text{ mol}$



* Proporción de cada reactivo:

$\text{Zn}: \frac{0.306}{1} = 0.306$; $\text{HCl}: \frac{1.2}{2} = 0.6$

* Limitante: Zn

* Moles de HCl en exceso: $n = 1.2 - 0.306 \cdot 2 = 0.588 \text{ mol}$

* Masa de HCl en exceso: $m = n \cdot M = 0.588 \cdot 36.5 = \boxed{21.5 \text{ g HCl}}$

b) * Moles de H_2 desprendidos:

$n = 0.306 \text{ mol Zn} \cdot \frac{1 \text{ mol H}_2}{1 \text{ mol Zn}} = 0.306 \text{ mol H}_2$

* Volumen de H_2 desprendidos:

$V = \frac{n \cdot R \cdot T}{P} = \frac{0.306 \cdot 0.082 \cdot 300}{1} = \boxed{7.53 \text{ L H}_2}$

39) a) $n_T = n_1 + n_2 = \frac{10}{2} + \frac{40}{32} = 5 + 1.25 = \boxed{6.25 \text{ mol}}$

$H_2: n_1 = \frac{m_1}{M_1} = \frac{10}{2} = \boxed{5 \text{ mol}}$

$O_2: n_2 = \frac{m_2}{M_2} = \frac{40}{32} = \boxed{1.25 \text{ mol}}$



* Proporciones:

$H_2: \frac{5}{2} = 2.5 ; O_2: \frac{1.25}{1} = 1.25$

* Limitante: O_2

* Moléculas de agua:

$$N = 1.25 \text{ mol } O_2 \cdot \frac{2 \text{ mol } H_2O}{1 \text{ mol } O_2} \cdot \frac{6.022 \cdot 10^{23} \text{ moléculas}}{1 \text{ mol } H_2O} =$$

$$= 1.25 \cdot 2 \cdot 6.022 \cdot 10^{23} = \boxed{1.51 \cdot 10^{24} \text{ moléculas}}$$

c) * Moles de H_2 en exceso:

$n = 5 - 1.25 \cdot 2 = 2.5 \text{ mol } H_2$

* Átomos de H en exceso:

$$N = 2.5 \text{ mol } H_2 \cdot \frac{6.022 \cdot 10^{23} \text{ moléculas } H_2}{1 \text{ mol } H_2} \cdot \frac{2 \text{ átomos H}}{1 \text{ molécula } H_2} =$$

$$= \boxed{3.01 \cdot 10^{24} \text{ átomos H}}$$

(40) a) * Molaridad 1:

$$c_{M1} = \frac{36 \text{ g HCl}}{100 \text{ g disolución}} \cdot \frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}} \cdot \frac{1.17 \text{ g disol.}}{1 \text{ ml disol.}} \cdot \frac{1000 \text{ ml disol.}}{1 \text{ l disol.}} = \frac{36 \cdot 1.17 \cdot 1000}{100 \cdot 36.5} = 11.5 \text{ M}$$

* Molaridad 2:

$$c_{M2} = \frac{12 \text{ g HCl}}{100 \text{ g disolución}} \cdot \frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}} \cdot \frac{1.05 \text{ g disol.}}{1 \text{ ml disol.}} \cdot \frac{1000 \text{ ml disol.}}{1 \text{ l disol.}} = \frac{12 \cdot 1.05 \cdot 1000}{100 \cdot 36.5} = 3.45 \text{ M}$$

* Volumen de disolución concentrada:

$$c_{M1} \cdot V_1 = c_{M2} \cdot V_2 \Rightarrow V_1 = \frac{c_{M2} \cdot V_2}{c_{M1}} = \frac{3.45 \cdot 50}{11.5} = \boxed{15 \text{ ml}}$$

b) * Reacción: $2 \text{ HCl} + \text{Mg(OH)}_2 \rightarrow \text{MgCl}_2 + 2 \text{ H}_2\text{O}$

* Volumen de disolución de Mg(OH)_2 :

$$v_a \cdot c_{Ma} \cdot V_a = v_b \cdot c_{Mb} \cdot V_b$$

$$V_b = \frac{v_a \cdot c_{Ma} \cdot V_a}{v_b \cdot c_{Mb}} = \frac{1 \cdot 3.45 \cdot 25}{2 \cdot 0.5} = \boxed{86.3 \text{ ml}}$$

41) a) $M_{PbS} = 239$

* Masa de PbS :

$$m = 0.41 \text{ l } \cancel{H_2S} \cdot \frac{1 \text{ mol } \cancel{H_2S}}{22.4 \text{ l } \cancel{H_2S}} \cdot \frac{1 \text{ mol } \cancel{PbS}}{1 \text{ mol } \cancel{H_2S}} \cdot \frac{239 \text{ g } PbS}{1 \text{ mol } \cancel{PbS}} =$$

$$= \frac{0.41 \cdot 239}{22.4} = 4.38 \text{ g } PbS$$

* Riqueza en PbS :

$$Riqueza = \frac{m_{PbS} \cdot 100}{m_T} = \frac{4.38 \cdot 100}{5} = \boxed{87.6\%}$$

b) * Volumen de H_2SO_4 :

$$V = 0.41 \text{ l } \cancel{H_2S} \cdot \frac{1 \text{ mol } \cancel{H_2S}}{22.4 \text{ l } \cancel{H_2S}} \cdot \frac{1 \text{ mol } \cancel{H_2SO_4}}{1 \text{ mol } \cancel{H_2S}} \cdot$$

$$\cdot \frac{1 \text{ l } \cancel{\text{disolución}}}{0.5 \text{ mol } \cancel{H_2SO_4}} \cdot \frac{1000 \text{ ml } \cancel{\text{disolución}}}{1 \text{ l } \cancel{\text{disolución}}} =$$

$$= \frac{0.41 \cdot 1000}{22.4 \cdot 0.5} = \boxed{36.6 \text{ ml}}$$

42) a) * Fracción molar:

$$X = \frac{n_s}{n_s + n_d} = \frac{\frac{37}{36.5}}{\frac{37}{36.5} + \frac{63}{18}} = \boxed{0.225}$$

b) * Molaridad del HCl:

$$C_{Ma} = \frac{37 \text{ g HCl}}{100 \text{ g disol.}} \cdot \frac{1 \text{ mol HCl}}{36.5 \text{ g HCl}} \cdot \frac{1.19 \text{ g disol.}}{1 \text{ ml disol.}} \cdot \frac{1000 \text{ ml disol.}}{1 \text{ l disol.}} = \frac{37 \cdot 1.19 \cdot 1000}{100 \cdot 36.5} = 12.1 \text{ M}$$

* Reacción: $\text{HCl} + \text{NaOH} \rightarrow \text{NaCl} + \text{H}_2\text{O}$

* Volumen de disolución de HCl:

$$V_a \cdot C_{Ma} \cdot V_a = V_b \cdot C_{Mb} \cdot V_b$$

$$V_a = \frac{V_b \cdot C_{Mb} \cdot V_b}{V_a \cdot C_{Ma}} = \frac{1 \cdot 0.12 \cdot 600}{1 \cdot 12.1} = \boxed{5.95 \text{ ml}}$$

43) a) * Molaridad 1:

$$C_{M1} = \frac{98 \text{ g } \cancel{\text{HNO}_3}}{100 \text{ g } \cancel{\text{disolución}}} \cdot \frac{1 \text{ mol } \text{HNO}_3}{63 \text{ g } \cancel{\text{HNO}_3}} \cdot \frac{1'505 \text{ g } \cancel{\text{disol.}}}{1 \text{ ml } \cancel{\text{disol.}}}$$
$$\cdot \frac{1000 \text{ ml } \cancel{\text{disol.}}}{1 \text{ l } \cancel{\text{disol.}}} = \frac{98 \cdot 1'505 \cdot 1000}{100 \cdot 63} = 23'4 \text{ M}$$

* Volumen necesario de ácido concentrado:

$$C_{M1} \cdot V_1 = C_{M2} \cdot V_2$$

$$V_1 = \frac{C_{M2} \cdot V_2}{C_{M1}} = \frac{1 \cdot 250}{23'4} = \boxed{10'7 \text{ ml}}$$

b) * Molaridad 3:

$$C_{M2} \cdot V_2 = C_{M3} \cdot V_3$$

$$C_{M3} = \frac{C_{M2} \cdot V_2}{V_3} = \frac{1 \cdot 50}{1000} = 0'05 \text{ M } \text{HNO}_3$$

* Reacción: $\text{HNO}_3 + \text{KOH} \rightarrow \text{KNO}_3 + \text{H}_2\text{O}$

$$M_{\text{KOH}} = 56'1$$

* Masa de KOH necesaria:

$$m = \frac{0'05 \text{ mol } \cancel{\text{HNO}_3}}{1 \cancel{\text{ l}}} \cdot 0'030 \cancel{\text{ l}} \cdot \frac{1 \text{ mol } \cancel{\text{KOH}}}{1 \text{ mol } \cancel{\text{HNO}_3}}$$

$$\cdot \frac{56'1 \text{ g } \text{KOH}}{1 \text{ mol } \cancel{\text{KOH}}} = \frac{0'05 \cdot 0'03 \cdot 56'1}{1} = \boxed{0'0842 \text{ g } \text{KOH}}$$

(44) a) * Molaridad 1:

$$C_{M1} = \frac{33'6 \text{ g } \cancel{\text{HNO}_3}}{100 \text{ g } \cancel{\text{disol.}}} \cdot \frac{1 \text{ mol } \text{HNO}_3}{63 \text{ g } \cancel{\text{HNO}_3}} \cdot \frac{1'5 \text{ g } \cancel{\text{disol.}}}{1 \text{ ml } \cancel{\text{disol.}}}$$

$$\cdot \frac{1000 \text{ ml } \cancel{\text{disol.}}}{1 \text{ l } \cancel{\text{disol.}}} = \frac{33'6 \cdot 1'5 \cdot 1000}{100 \cdot 63} = 8 \text{ M}$$

* Volumen de ácido concentrado:

$$C_{M1} \cdot V_1 = C_{M2} \cdot V_2$$

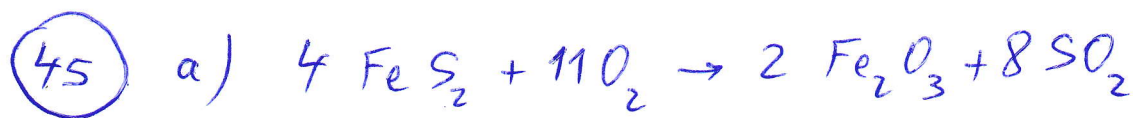
$$V_1 = \frac{C_{M2} \cdot V_2}{C_{M1}} = \frac{0'2 \cdot 1000}{8} = \boxed{25 \text{ ml}}$$

b) * Reacción: $2\text{HNO}_3 + \text{Ca}(\text{OH})_2 \rightarrow \text{Ca}(\text{NO}_3)_2 + 2\text{H}_2\text{O}$

* Molaridad de la disolución de $\text{Ca}(\text{OH})_2$:

$$V_a \cdot C_{Ma} \cdot V_a = V_b \cdot C_{Mb} \cdot V_b$$

$$C_{Mb} = \frac{V_a \cdot C_{Ma} \cdot V_a}{V_b \cdot V_b} = \frac{1 \cdot 0'2 \cdot 40}{2 \cdot 20} = \boxed{0'2 \text{ M}}$$



b) $M_{\text{H}_2\text{SO}_4} = 98$; $M_{\text{FeS}_2} = 119.9$

* Masa de H_2SO_4 :

$$m = 10^6 \text{ g pirita} \cdot \frac{90 \text{ g } \cancel{\text{FeS}_2}}{100 \text{ g pirita}} \cdot \frac{1 \text{ mol } \cancel{\text{FeS}_2}}{119.9 \text{ g } \cancel{\text{FeS}_2}} \cdot \frac{8 \text{ mol } \cancel{\text{SO}_2}}{4 \text{ mol } \cancel{\text{FeS}_2}} \cdot \frac{2 \text{ mol } \cancel{\text{SO}_3}}{2 \text{ mol } \cancel{\text{SO}_2}} \cdot \frac{1 \text{ mol } \text{H}_2\text{SO}_4}{1 \text{ mol } \cancel{\text{SO}_3}} \cdot \frac{98 \text{ g } \cancel{\text{H}_2\text{SO}_4}}{1 \text{ mol } \text{H}_2\text{SO}_4} \cdot \frac{1 \text{ kg } \text{H}_2\text{SO}_4}{1000 \text{ g } \cancel{\text{H}_2\text{SO}_4}} = \frac{10^6 \cdot 90 \cdot 8 \cdot 98}{100 \cdot 119.9 \cdot 4 \cdot 1000} =$$

$$= \boxed{1470 \text{ kg } \text{H}_2\text{SO}_4}$$

c) * Masa de Fe:

$$m = 10^6 \text{ g pirita} \cdot \frac{90 \text{ g } \cancel{\text{FeS}_2}}{100 \text{ g pirita}} \cdot \frac{1 \text{ mol } \cancel{\text{FeS}_2}}{119.9 \text{ g } \cancel{\text{FeS}_2}} \cdot \frac{2 \text{ mol } \cancel{\text{Fe}_2\text{O}_3}}{4 \text{ mol } \cancel{\text{FeS}_2}} \cdot \frac{4 \cdot 55.85 \text{ g } \cancel{\text{Fe}}}{2 \text{ mol } \cancel{\text{Fe}_2\text{O}_3}} \cdot \frac{1 \text{ kg } \text{Fe}}{1000 \text{ g } \cancel{\text{Fe}}} \cdot \frac{80}{100} =$$

$$= \frac{10^6 \cdot 90 \cdot 2 \cdot 4 \cdot 55.85 \cdot 80}{100 \cdot 119.9 \cdot 4 \cdot 2 \cdot 1000 \cdot 100} = \boxed{335 \text{ kg Fe}}$$

(210)

46) $M_{H_2S} = 34$; $M_{Cr_2(SO_4)_3} = 392$

a) * Moles de H_2S : $n = \frac{m}{M} = \frac{40}{34} = 1.18 \text{ mol } H_2S$

* Masa de $Cr_2(SO_4)_3$:

$$m = 1.18 \text{ mol } H_2S \cdot \frac{1 \text{ mol } Cr_2(SO_4)_3}{3 \text{ mol } H_2S} \cdot \frac{392 \text{ g } Cr_2(SO_4)_3}{1 \text{ mol } Cr_2(SO_4)_3} =$$

$$= \frac{1.18 \cdot 392}{3} = \boxed{154 \text{ g } Cr_2(SO_4)_3}$$

b) * Moles de S:

$$n = 1.18 \text{ mol } H_2S \cdot \frac{8 \text{ mol S}}{8 \text{ mol } H_2S} = \boxed{1.18 \text{ mol S}}$$

c) * Volumen de agua líquida:

$$V = 1.18 \text{ mol } H_2S \cdot \frac{7 \text{ mol } H_2O}{3 \text{ mol } H_2S} \cdot \frac{18 \text{ g } H_2O}{1 \text{ mol } H_2O} \cdot \frac{1 \text{ ml } H_2O}{1 \text{ g } H_2O} =$$

$$= \frac{1.18 \cdot 7 \cdot 18}{3} = \boxed{49.6 \text{ ml } H_2O}$$

d) * Moles de agua:

$$n = 1.18 \text{ mol } H_2S \cdot \frac{7 \text{ mol } H_2O}{3 \text{ mol } H_2S} = 2.75 \text{ mol } H_2O$$

* Volumen de agua gaseosa:

$$V = \frac{n \cdot R \cdot T}{P} = \frac{2.75 \cdot 0.082 \cdot 303}{2} = \boxed{34.2 \text{ l } H_2O}$$

$$(47) \quad M_{\text{NaHSO}_3} = 104 \quad ; \quad M_{\text{NaHSO}_4} = 120$$

* Moles de NaHSO_3 puro:

$$n = 70 \text{ g producto} \cdot \frac{80 \text{ g } \cancel{\text{NaHSO}_3}}{100 \text{ g producto}} \cdot \frac{1 \text{ mol } \cancel{\text{NaHSO}_3}}{104 \text{ g } \cancel{\text{NaHSO}_3}} =$$

$$= \frac{70 \cdot 80}{100 \cdot 104} = 0.539 \text{ mol } \text{NaHSO}_3$$

a) * Masa de NaHSO_4 :

$$m = 0.539 \text{ mol } \cancel{\text{NaHSO}_3} \cdot \frac{3 \text{ mol } \cancel{\text{NaHSO}_4}}{5 \text{ mol } \cancel{\text{NaHSO}_3}} \cdot \frac{120 \text{ g } \cancel{\text{NaHSO}_4}}{1 \text{ mol } \cancel{\text{NaHSO}_4}} =$$

$$= \frac{0.539 \cdot 3 \cdot 120}{5} = \boxed{38.8 \text{ g}}$$

b) * Moles de Na_2SO_4 :

$$n = 0.539 \text{ mol } \cancel{\text{NaHSO}_3} \cdot \frac{2 \text{ mol } \cancel{\text{Na}_2\text{SO}_4}}{5 \text{ mol } \cancel{\text{NaHSO}_3}} = \boxed{0.216 \text{ mol } \text{Na}_2\text{SO}_4}$$

c) * Moléculas de I_2 :

$$N = 0.539 \text{ mol } \cancel{\text{NaHSO}_3} \cdot \frac{1 \text{ mol } \cancel{\text{I}_2}}{5 \text{ mol } \cancel{\text{NaHSO}_3}} \cdot \frac{6.022 \cdot 10^{23} \text{ moléculas } \cancel{\text{I}_2}}{1 \text{ mol } \cancel{\text{I}_2}} =$$

$$= \boxed{6.49 \cdot 10^{22} \text{ moléculas } \text{I}_2}$$

d) * Volumen de agua gaseosa:

$$V = 0.539 \text{ mol } \cancel{\text{NaHSO}_3} \cdot \frac{1 \text{ mol } \cancel{\text{H}_2\text{O}}}{5 \text{ mol } \cancel{\text{NaHSO}_3}} \cdot \frac{22.4 \text{ l } \cancel{\text{H}_2\text{O}}}{1 \text{ mol } \cancel{\text{H}_2\text{O}}} =$$

$$= \boxed{2.42 \text{ l } \text{H}_2\text{O}}$$

(212)

48) $M_{KMnO_4} = 157'9$; $M_{MnSO_4} = 150'9$

a) * Moles de $KMnO_4$: $n = \frac{m}{M} = \frac{90}{157'9} = 0'57 \text{ mol}$

* Masa de $MnSO_4$:

$$m = 0'57 \text{ mol } \cancel{KMnO_4} \cdot \frac{2 \text{ mol } \cancel{MnSO_4}}{2 \text{ mol } \cancel{KMnO_4}} \cdot \frac{150'9 \text{ g } MnSO_4}{1 \text{ mol } \cancel{MnSO_4}} =$$

$$= 0'57 \cdot 150'9 = \boxed{86 \text{ g } MnSO_4}$$

b) * Moles de H_2O :

$$n = 0'57 \text{ mol } \cancel{KMnO_4} \cdot \frac{8 \text{ mol } H_2O}{2 \text{ mol } \cancel{KMnO_4}} = \boxed{2'28 \text{ mol } H_2O}$$

c) * Moléculas de K_2SO_4 :

$$N = 0'57 \text{ mol } \cancel{KMnO_4} \cdot \frac{1 \text{ mol } \cancel{K_2SO_4}}{2 \text{ mol } \cancel{KMnO_4}} \cdot \frac{6'022 \cdot 10^{23} \text{ moléculas}}{1 \text{ mol } \cancel{K_2SO_4}} =$$

$$= \frac{0'57 \cdot 6'022 \cdot 10^{23}}{2} = \boxed{1'72 \cdot 10^{23} \text{ moléculas } K_2SO_4}$$

d) * Moles de O_2 :

$$n = 0'57 \text{ mol } \cancel{KMnO_4} \cdot \frac{5 \text{ mol } O_2}{2 \text{ mol } \cancel{KMnO_4}} = 1'43 \text{ mol } O_2$$

* Volumen de O_2 :

$$V = \frac{n \cdot R \cdot T}{P} = \frac{1'43 \cdot 0'082 \cdot (273 + 80)}{0'5} = \boxed{82'8 \text{ l } O_2}$$

$$(49) \quad M_{\text{MnSO}_4} = 150'9 \quad ; \quad M_{\text{FeSO}_4} = 151'9$$

a) * Moles de FeSO_4 puro:

$$n = 120 \text{ g mineral} \cdot \frac{90 \text{ g FeSO}_4}{100 \text{ g mineral}} \cdot \frac{1 \text{ mol FeSO}_4}{151'9 \text{ g FeSO}_4} =$$

$$= \frac{120 \cdot 90}{100 \cdot 151'9} = 0'711 \text{ mol FeSO}_4$$

* Masa de MnSO_4 :

$$m = 0'711 \text{ mol FeSO}_4 \cdot \frac{2 \text{ mol MnSO}_4}{10 \text{ mol FeSO}_4} \cdot \frac{150'9 \text{ g MnSO}_4}{1 \text{ mol MnSO}_4} \cdot \frac{75}{100} =$$

$$= \frac{0'711 \cdot 2 \cdot 150'9 \cdot 75}{10 \cdot 100} = \boxed{16'1 \text{ g MnSO}_4}$$

b) * Masa de mineral:

$$m = 10^5 \text{ g MnSO}_4 \cdot \frac{1 \text{ mol MnSO}_4}{150'9 \text{ g MnSO}_4} \cdot \frac{10 \text{ mol FeSO}_4}{2 \text{ mol MnSO}_4} \cdot$$

$$\cdot \frac{151'9 \text{ g FeSO}_4}{1 \text{ mol FeSO}_4} \cdot \frac{100 \text{ g reales FeSO}_4}{75 \text{ g técnicos}} \cdot \frac{100 \text{ g mineral}}{90 \text{ g FeSO}_4} \cdot$$

$$\cdot \frac{1 \text{ kg mineral}}{1000 \text{ g mineral}} = \frac{10^5 \cdot 10 \cdot 151'9 \cdot 100 \cdot 100}{150'9 \cdot 2 \cdot 75 \cdot 90 \cdot 1000} =$$

$$= \boxed{746 \text{ kg mineral}}$$

$$(50) \quad M_{K_2MnO_4} = 196'9 \quad ; \quad M_{KMnO_4} = 157'9$$

a) * Moles de $KMnO_4$ puro:

$$n = 120 \text{ g mineral} \cdot \frac{80 \text{ g } \cancel{KMnO_4}}{100 \text{ g mineral}} \cdot \frac{1 \text{ mol } KMnO_4}{157'9 \text{ g } \cancel{KMnO_4}} =$$

$$= \frac{120 \cdot 80}{100 \cdot 157'9} = 0'608 \text{ mol } KMnO_4$$

* Masa de K_2MnO_4 :

$$m = 0'608 \text{ mol } \cancel{KMnO_4} \cdot \frac{8 \text{ mol } \cancel{K_2MnO_4}}{8 \text{ mol } \cancel{KMnO_4}} \cdot \frac{196'9 \text{ g } K_2MnO_4}{1 \text{ mol } \cancel{K_2MnO_4}} \cdot \frac{65}{100} = \frac{0'608 \cdot 196'9 \cdot 65}{100} = \boxed{77'8 \text{ g } K_2MnO_4}$$

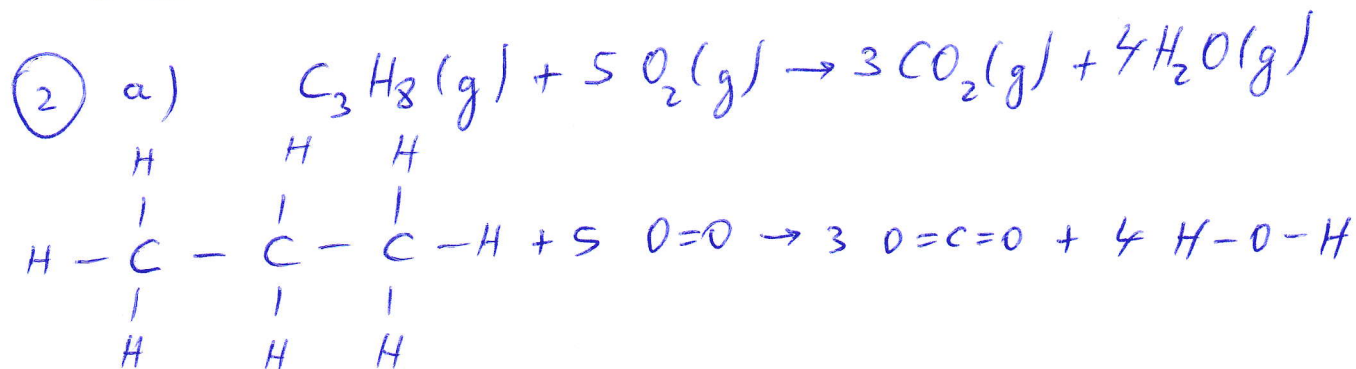
b) * Masa de mineral:

$$m = 100 \text{ g } \cancel{K_2MnO_4} \cdot \frac{1 \text{ mol } \cancel{K_2MnO_4}}{196'9 \text{ g } \cancel{K_2MnO_4}} \cdot \frac{8 \text{ mol } \cancel{KMnO_4}}{8 \text{ mol } \cancel{K_2MnO_4}} \cdot \frac{157'9 \text{ g } \cancel{KMnO_4}}{1 \text{ mol } \cancel{KMnO_4}} \cdot \frac{100 \text{ g reales } KMnO_4}{65 \text{ g teóricos}} \cdot \frac{100 \text{ g mineral}}{80 \text{ g } \cancel{KMnO_4}} =$$

$$= \frac{100 \cdot 157'9 \cdot 100 \cdot 100}{196'9 \cdot 65 \cdot 80} = \boxed{154 \text{ g mineral}}$$

PROBLEMAS RESUELTOS DE TERMOQUÍMICA

$$\begin{aligned}
 \textcircled{1} \quad \Delta H_R &= \sum (n \cdot H)_{\text{productos}} - \sum (n \cdot H)_{\text{reactivos}} = \\
 &= 16 \cdot (-393'5) + 18 \cdot (-285'84) - 2 \cdot (-268) - 25 \cdot 0 = \\
 &= \boxed{-1'09 \cdot 10^4 \text{ kJ}}
 \end{aligned}$$



$$\begin{aligned}
 \Delta H_c &= \sum EE_{\text{rotos}} - \sum EE_{\text{formados}} = \\
 &= 2 \cdot E_{\text{C-C}} + 8 \cdot E_{\text{C-H}} + 5 \cdot E_{\text{O=O}} - 3 \cdot 2 \cdot E_{\text{C=O}} - 4 \cdot 2 \cdot E_{\text{O-H}} = \\
 &= 2 \cdot 347 + 8 \cdot 415 + 5 \cdot 494 - 6 \cdot 730 - 8 \cdot 460 = \\
 &= \boxed{-1576 \text{ kJ}}
 \end{aligned}$$

b) * Moles de CO_2 :

$$n = 5990 \text{ kJ} \cdot \frac{3 \text{ mol CO}_2}{1576 \text{ kJ}} = 11'4 \text{ mol CO}_2$$

* Volumen de CO_2 :

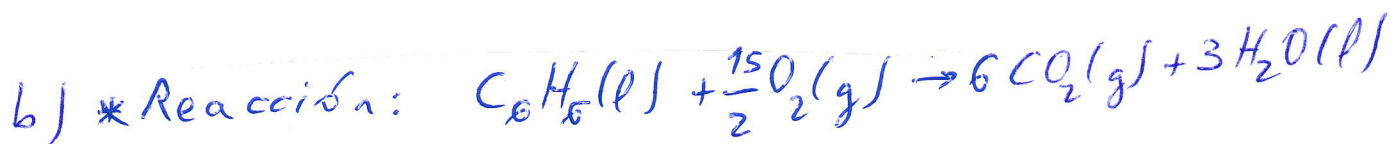
$$V = \frac{n \cdot R \cdot T}{p} = \frac{11'4 \cdot 0'082 \cdot 298}{1} = \boxed{279 \text{ l CO}_2}$$

③ $M_{C_6H_6} = 78$

a) * Moles de C_6H_6 : $n = \frac{m}{M} = \frac{235}{78} = 0.0301 \text{ mol}$

* Calor a volumen constante, ΔU :

$$\Delta U = \frac{Q}{n} = \frac{-9853}{0.0301} = \boxed{-3273 \frac{\text{kJ}}{\text{mol}}}$$



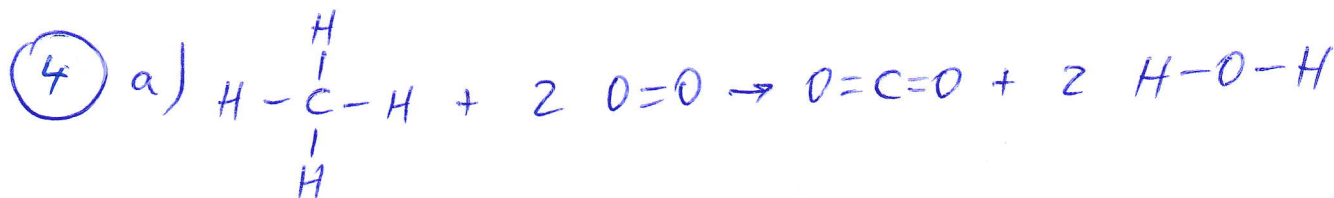
* Incremento de moles gaseosos:

$$\Delta n = 6 - \frac{15}{2} = \frac{12-15}{2} = -\frac{3}{2} = -1.5 \text{ mol}$$

* Calor a presión constante, ΔH :

$$\Delta U = \Delta H - R \cdot T \cdot \Delta n \Rightarrow \Delta H = \Delta U + R \cdot T \cdot \Delta n =$$

$$= -3273 + 8.31 \cdot 10^{-3} \cdot 298 \cdot (-1.5) = \boxed{-3277 \frac{\text{kJ}}{\text{mol}}}$$



$$\Delta H = \sum EE_{\text{rotos}} - \sum EE_{\text{formados}} =$$

$$= 4 \cdot E_{C-H} + 2 \cdot E_{O=O} - 2 \cdot E_{C=O} - 2 \cdot 2 \cdot E_{O-H} =$$

$$= 4 \cdot 414 + 2 \cdot 498 - 2 \cdot 707 - 4 \cdot 464 = \boxed{-618 \text{ kJ}}$$

b) * Moles de O_2 : $n = \frac{P \cdot V}{R \cdot T} = \frac{1 \cdot 10^5}{0.082 \cdot 398} = 0.322 \text{ mol } O_2$

* Energía desprendida:

$$E = \frac{-618 \text{ kJ}}{2 \text{ mol } O_2} \cdot 0.322 \text{ mol } O_2 = (217) = \boxed{-99.5 \text{ kJ}}$$

$$(5) \quad a) \quad \Delta U = \Delta H - R \cdot T \cdot \Delta n$$

$$\Delta n = 2 - 1 - 3 = 2 - 4 = -2$$

$$\Delta U = -92'4 - 8'31 \cdot 10^{-3} \cdot 298 \cdot (-2) = \boxed{-87'4 \text{ kJ}}$$

$$b) \quad \Delta S = \sum (n \cdot S)_{\text{productos}} - \sum (n \cdot S)_{\text{reactivos}} =$$

$$= 2 \cdot 192'3 - 1 \cdot 191 - 3 \cdot 130'8 = -199 \frac{\text{J}}{\text{K}}$$

$$\Delta G = \Delta H - T \cdot \Delta S = -92'4 - 298 \cdot (-199) \cdot 10^{-3} =$$

$$= \boxed{-33'1 \text{ kJ}}$$

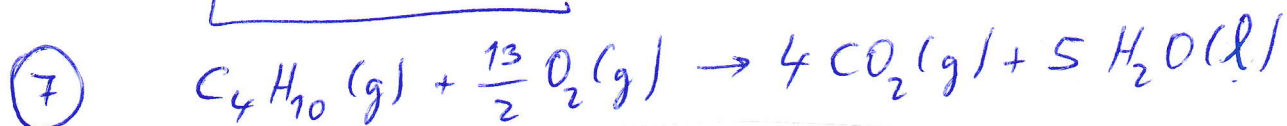
$$(6) \quad \Delta H = \sum (n \cdot H)_{\text{productos}} - \sum (n \cdot H)_{\text{reactivos}} =$$

$$= -393'52 + 2 \cdot (-285'84) - (-74'87) - 2 \cdot 0 = -890 \frac{\text{kJ}}{\text{mol}}$$

$$\Delta n = 1 - 1 - 2 = -2$$

$$\Delta U = \Delta H - R \cdot T \cdot \Delta n = -890 - 8'31 \cdot 10^{-3} \cdot 298 \cdot (-2) =$$

$$= \boxed{-885 \text{ kJ}}$$



$$\Delta n = 4 - 1 - \frac{13}{2} = 3 - \frac{13}{2} = -\frac{7}{2}$$

$$\Delta U = \Delta H - R \cdot T \cdot \Delta n = -2877'3 - 8'31 \cdot 10^{-3} \cdot 298 \cdot \left(-\frac{7}{2}\right) =$$

$$= \boxed{-2869 \text{ kJ}}$$

$$\textcircled{8} \quad \Delta S = \sum (n \cdot S)_{\text{productos}} - \sum (n \cdot S)_{\text{reactivos}} =$$

$$= 2 \cdot 192,3 - 191 - 3 \cdot 130,8 = \boxed{-199 \frac{\text{J}}{\text{mol} \cdot \text{K}}}$$

$\textcircled{9}$ Si el Δn de moles gaseosos es positivo, ΔS es positivo. Si $\Delta n < 0 \Rightarrow \Delta S < 0$.

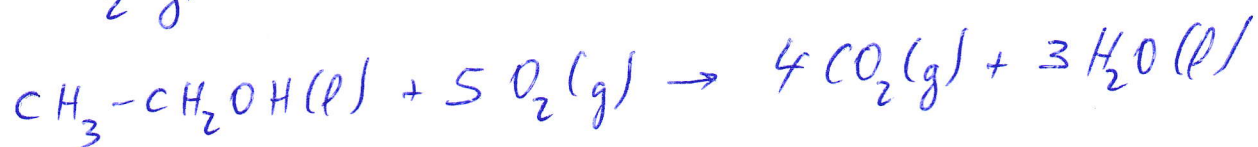
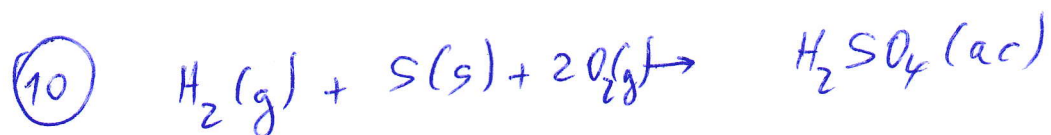
$$a) \quad \Delta n = 2 - 2 - 1 = -1 \Rightarrow \Delta S < 0$$

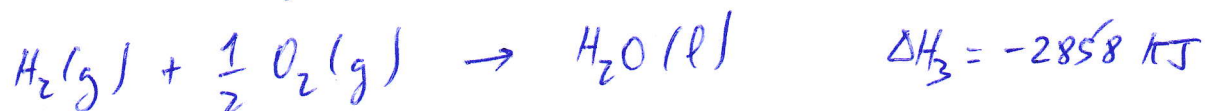
$$b) \quad \Delta n = 1 + 2 - 1 - 2 = 0 \Rightarrow \Delta S = 0$$

$$c) \quad \Delta n = 8 + 9 - 1 - \frac{25}{2} = 16 - \frac{25}{2} = \frac{32 - 25}{2} = \frac{7}{2} \Rightarrow \Delta S > 0$$

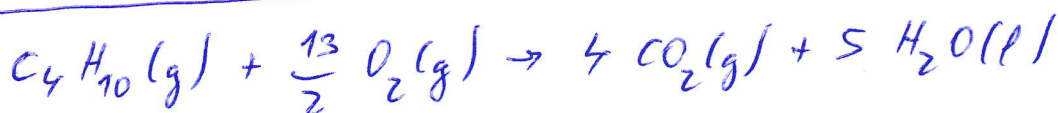
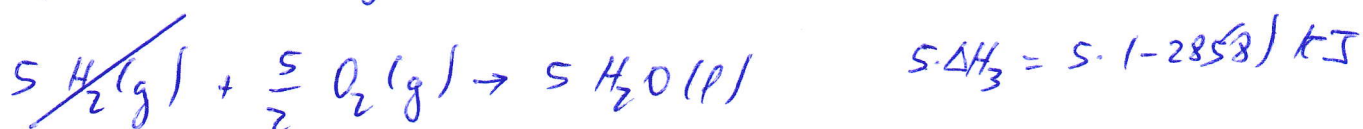
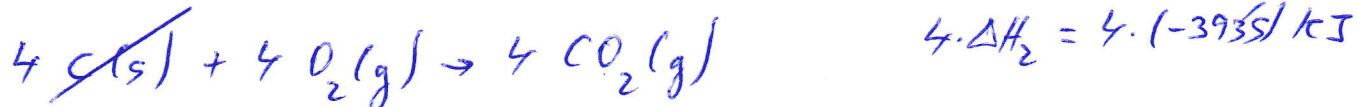
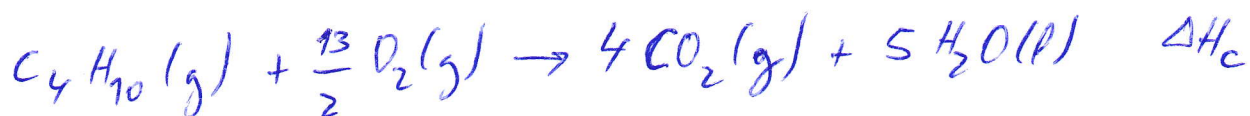
$$d) \quad \Delta n = 0 - 1 - 3 - 1 = -5 \Rightarrow \Delta S < 0$$

$$e) \quad \Delta n = 1 + 1 = 2 \Rightarrow \Delta S > 0$$





Combustion:



$$\Delta H_c = -\Delta H_1 + 4 \cdot \Delta H_2 + 5 \cdot \Delta H_3 =$$

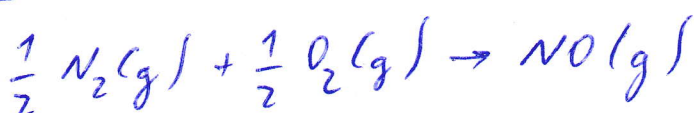
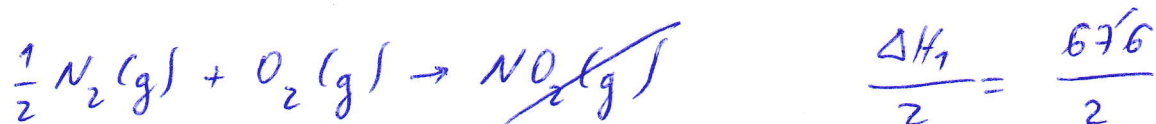
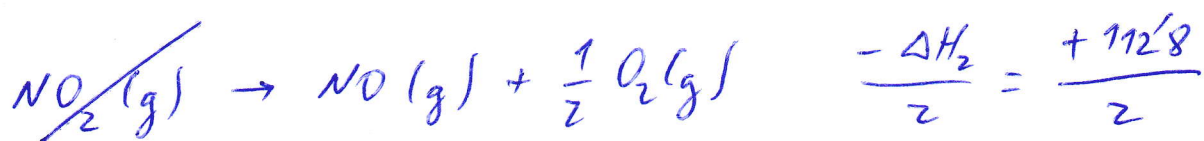
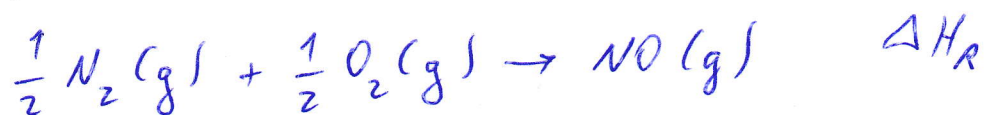
$$= 1257 - 4 \cdot 3935 - 5 \cdot 2858 = \boxed{-2877 \frac{\text{kJ}}{\text{mol}}}$$

$$\text{b) } \Delta U = \Delta H - R \cdot T \cdot \Delta n$$

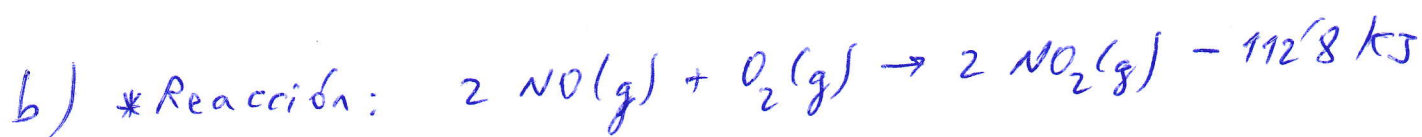
$$\Delta n = 4 - 1 - \frac{13}{2} = 3 - \frac{13}{2} = -\frac{7}{2}$$

$$\Delta U = -2877 - 8.31 \cdot 10^{-3} \cdot 298 \cdot \left(-\frac{7}{2}\right) = \boxed{-2868 \frac{\text{kJ}}{\text{mol}}}$$

12) a) * Formación de NO:



$$\Delta H_R = \frac{\Delta H_1}{2} - \frac{\Delta H_2}{2} = \frac{67'6}{2} + \frac{112'8}{2} = \boxed{90'2 \frac{\text{kJ}}{\text{mol}}}$$



* Litros de aire:

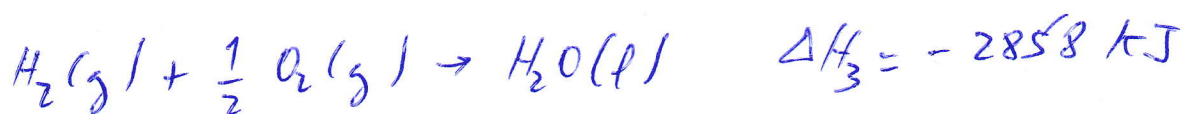
$$V = 50 \text{ L } \cancel{\text{NO}} \cdot \frac{1 \text{ L } \cancel{\text{O}_2}}{2 \text{ L } \cancel{\text{NO}}} \cdot \frac{100 \text{ L aire}}{21 \text{ L } \cancel{\text{O}_2}} = \boxed{119 \text{ L aire}}$$

* Moles de NO:

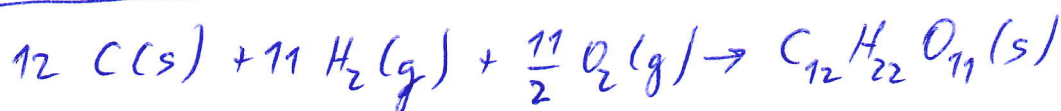
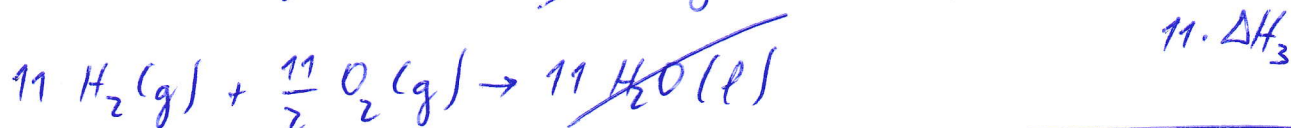
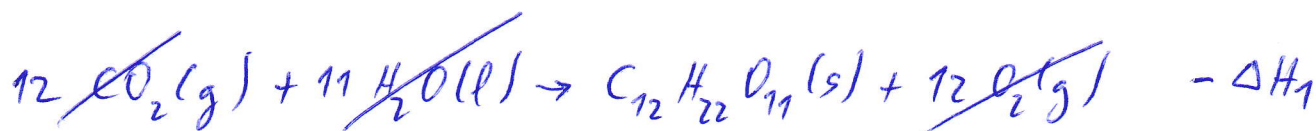
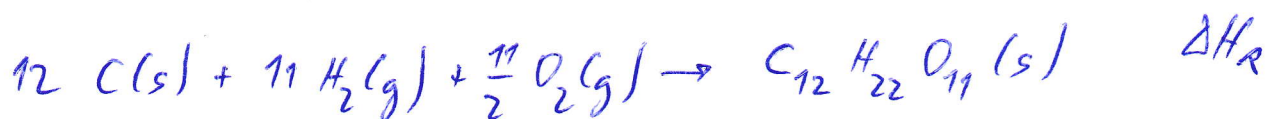
$$n = \frac{P \cdot V}{R \cdot T} = \frac{1 \cdot 50}{0'082 \cdot 298} = 2'05 \text{ mol NO}$$

* Calor en juego:

$$Q = \frac{-112'8 \text{ kJ}}{2 \text{ mol } \cancel{\text{NO}}} \cdot 2'05 \text{ mol } \cancel{\text{NO}} = \boxed{-116 \text{ kJ}}$$



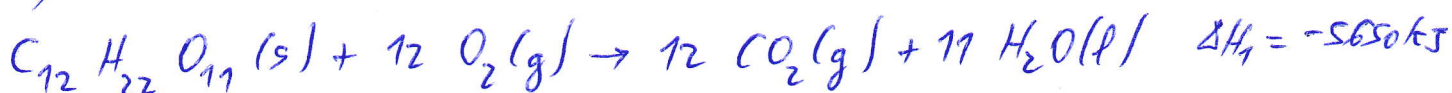
* Reacción:



$$\Delta H_R = -\Delta H_1 + 12 \cdot \Delta H_2 + 11 \cdot \Delta H_3 =$$

$$= 5650 + 12 \cdot (-393.5) + 11 \cdot (-285.8) = \boxed{-2216 \frac{\text{kJ}}{\text{mol}}}$$

b) * Combustión:

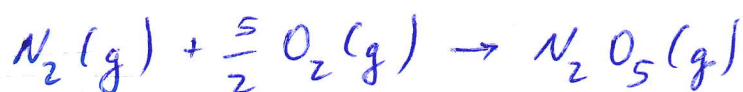
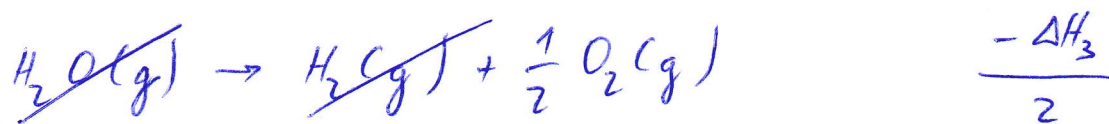
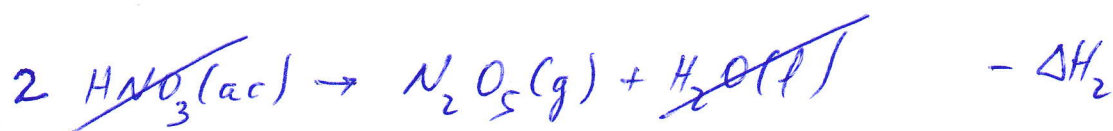
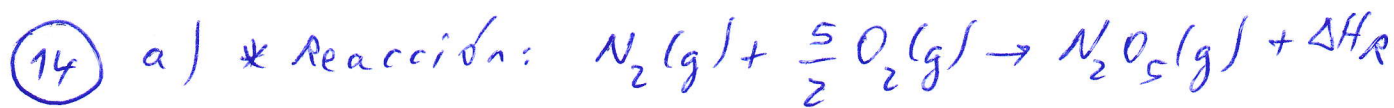


$$M_{C_{12}H_{22}O_{11}} = 342$$

* Trabajo muscular:

$$W = \frac{-5650 \text{ kJ}}{1 \text{ mol sacarosa}} \cdot \frac{30}{100} \cdot \frac{1 \text{ mol sacarosa}}{342 \text{ g sacarosa}} \cdot 1 \text{ g sacarosa} =$$

$$= \boxed{4.96 \text{ kJ}} \quad (222)$$



$$\Delta H_4 = 540 \frac{\cancel{cal}}{\cancel{g}} \cdot \frac{418 \cancel{J}}{1 \cancel{cal}} \cdot \frac{1 \text{ kJ}}{1000 \cancel{J}} \cdot \frac{18 \cancel{g}}{1 \text{ mol}} =$$

$$= \frac{540 \cdot 418 \cdot 18}{1000} = 40'6 \frac{\text{kJ}}{\text{mol}}$$

$$\Delta H_R = \Delta H_1 - \Delta H_2 - \frac{\Delta H_3}{2} + \Delta H_4 =$$

$$= -414'7 + 140'20 + \frac{571'70}{2} + 40'6 = \boxed{51'95 \frac{\text{kJ}}{\text{mol}}}$$

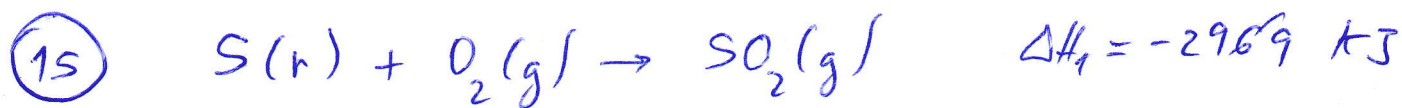
b) * Moles de N_2O_5 :

$$n = \frac{P \cdot V}{R \cdot T} = \frac{1 \cdot 50}{0'082 \cdot 298} = 2'05 \text{ mol } N_2O_5$$

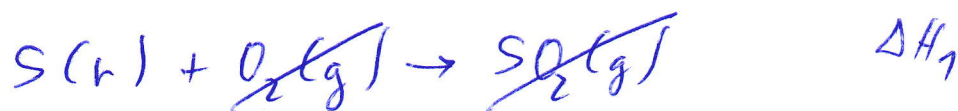
* Energía necesaria:

$$Q = \frac{51'95 \text{ kJ}}{1 \text{ mol } \cancel{N_2O_5}} \cdot 2'05 \text{ mol } \cancel{N_2O_5} = \boxed{106 \text{ kJ}}$$

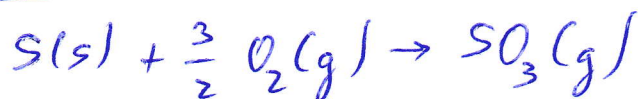
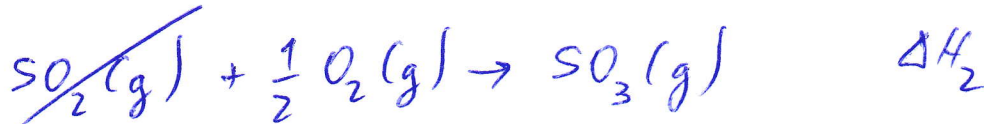
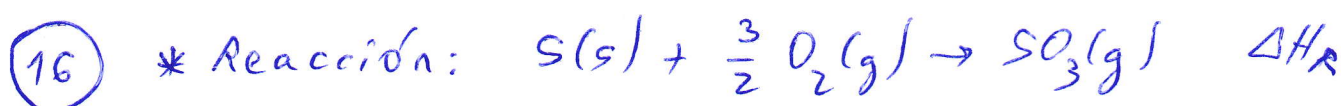
(223)



* Reacción: $S(r) \rightarrow S(m)$



$\Delta H_R = \Delta H_1 - \Delta H_2 = -296'9 + 296'56 = \boxed{-0'34 \frac{\text{kJ}}{\text{mol}}}$



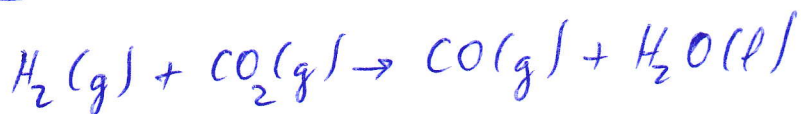
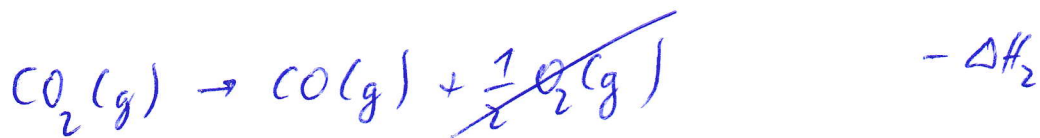
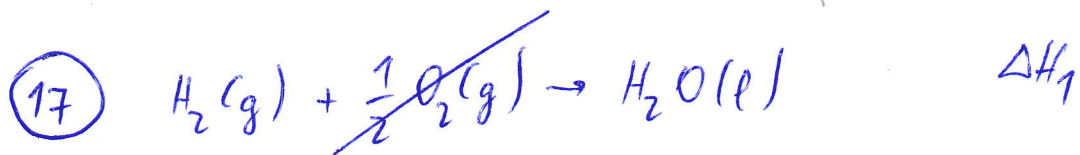
$\Delta H_R = \Delta H_1 + \Delta H_2 = -297'1 - 98'28 = -395 \frac{\text{kJ}}{\text{mol}}$

* Moles de SO_3 :

$n = \frac{P \cdot V}{R \cdot T} = \frac{1 \cdot 10}{0'082 \cdot 298} = 0'409 \text{ mol } SO_3$

* Calor desprendido:

$Q = \frac{-395 \text{ kJ}}{1 \text{ mol } \cancel{SO_3}} \cdot 0'409 \text{ mol } \cancel{SO_3} = \boxed{-162 \text{ kJ}}$

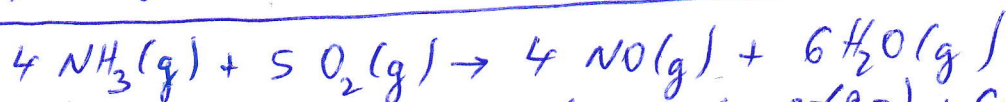
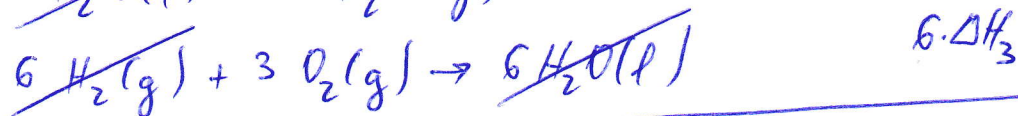
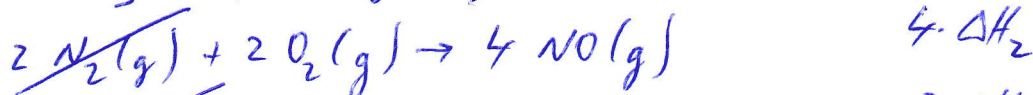
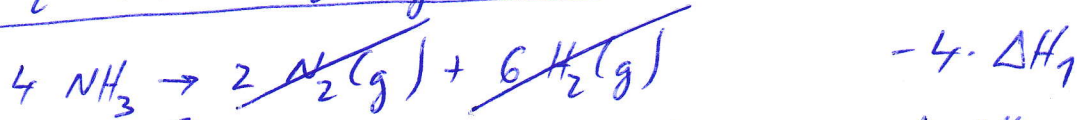
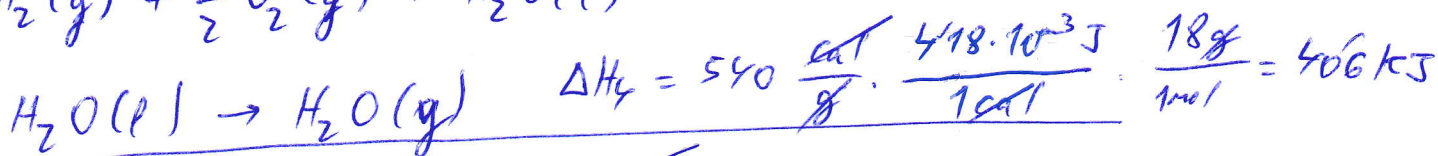
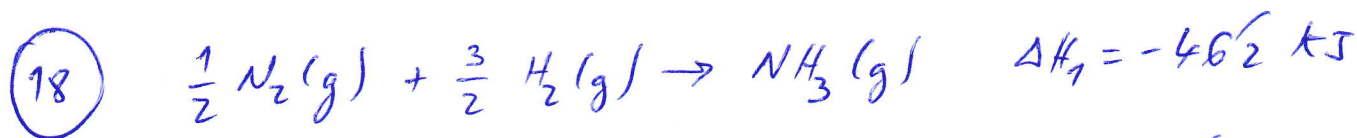


$$\Delta H_R = \Delta H_1 - \Delta H_2 = -285'76 + 282'83 = -2'93 \frac{\text{kJ}}{\text{mol}}$$

$$M_{\text{CO}} = 12 + 16 = 28 \frac{\text{g}}{\text{mol}}$$

* Calor desprendido:

$$Q = \frac{-2'93 \text{ kJ}}{1 \text{ mol CO}} \cdot \frac{1 \text{ mol CO}}{28 \text{ g CO}} \cdot 20 \text{ g CO} = \boxed{-2'09 \text{ kJ}}$$



$$\Delta H_R = -4 \cdot (-46'2) + 4 \cdot 90'4 + 6 \cdot (-285'85) + 6 \cdot 40'6 =$$

$$= \boxed{-925 \text{ kJ}} \quad (225)$$



$$\Delta H_R = -2 \cdot \Delta H_1 + \Delta H_2 = -2 \cdot 11'29 - 2'93 = -25'5 \frac{\text{kJ}}{\text{mol}}$$

* Moles de CO_2 :

$$n = \frac{P \cdot V}{R \cdot T} = \frac{1 \cdot 30}{0.082 \cdot 298} = 1'23 \text{ mol CO}_2$$

* Calor desprendido:

$$Q = \frac{-25'5 \text{ kJ}}{3 \text{ mol CO}_2} \cdot 1'23 \text{ mol CO}_2 = \boxed{-10'5 \text{ kJ}}$$

$$(20) \Delta S^\circ = \sum (n \cdot S)_{\text{productos}} - \sum (n \cdot S)_{\text{reactivos}} =$$

$$= 2 \cdot 192'3 - 191 - 3 \cdot 130'8 = -198'8 \frac{\text{J}}{\text{mol} \cdot \text{K}} = -0'1988 \frac{\text{kJ}}{\text{mol} \cdot \text{K}}$$

$$\Delta G^\circ = \Delta H^\circ - T \cdot \Delta S^\circ = -46'11 - 298 \cdot (-0'1988) =$$

$$= +13'1 \text{ kJ}$$

Al ser $\Delta G^\circ > 0$, no es espontánea.

$$(21) \quad \Delta H = -4800 \text{ kJ} \quad ; \quad \Delta S = -6'45 \text{ kJ}$$

$$\Delta G = \Delta H - T \cdot \Delta S = -4800 - T \cdot (-6'45) < 0 \Rightarrow$$

$$\Rightarrow +6'45 \cdot T < 4800 \Rightarrow$$

$$T < \frac{4800}{6'45} = 744 \text{ K} = \boxed{471^\circ \text{C}}$$

Una reacción es espontánea cuando su $\Delta G < 0$.